1. (a) Devise a recursive algorithm to find \(a^{2^n}\), where \(a\) is a real number and \(n\) is a positive integer. [Hint: Use the equality \(a^{2^{n+1}} = (a^{2^n})^2\).]

(b) Give a recursive algorithm for finding the sum of the first \(n\) positive integers.

2. Let \(x\) and \(y\) be real numbers. Consider the program

\[
\text{if } x < y \\
\quad \text{min} = x \\
\text{else} \\
\quad \text{min} = y
\]

Prove that \(((x \leq y) \land min = x) \lor ((x > y) \land min = y)\) is true after this code is executed. Be careful to consider the case where \(x = y\).

3. Give an example of a function from the set of integers to the set of integers that is

(a) one-to-one but not onto.
(b) onto but not one-to-one.
(c) both onto and one-to-one (but different from the identity function).
(d) neither one-to-one nor onto.

4. Give a recursive algorithm for finding a mode of a list of integers. (A mode is an element in the list that occurs at least as often as every other element.)