Announcements

- Quiz in one week
- Homework released Thursday
- Exam Thursday!
Exam 1 Review

- Exam 1 covers all of Chapter 1, Sets in Chapter 2, a little on induction

- Questions will be a mix of short answer, multiple choice (maybe)

- Be prepared to prove propositions using
  - Direct Proofs
  - Contradictions
  - Contrapositions

- Each step in a proof needs to clearly follow from the previous
  - Modus Tollens
  - Modus Ponens
  - definition of an odd
  - Simple algebra
  - Etc...
Exam 1 Review

• Exam 1 covers all of Chapter 1, Sets in Chapter 2, a little on induction

• Questions will be a mix of short answer, multiple choice

• Be prepared to prove properties of sets using
  – Set Tables
  – Equivalence relations

• Understand the basics of an induction proof
Exam 1 Review

• **Additional terms to be familiar with**
  - Natural Numbers
    - \{0, 1, 2, 3...\}
  - Integers
    - \{..., -2, -1, 0, 1, 2, ...
  - Rational Numbers
    - \{p/q where p and q are integers and q \neq 0\}
  - Real Numbers
    - Non imaginary numbers
  - Prime Numbers
    - Integers > 1 with no positive divisors besides 1 and itself \{2, 3, 5, 7, 11, and many more\}
  - Composite Numbers
    - Positive integers > 1 that are not prime

• **Proof Fallacies**
  - Affirming the conclusion
    - If you do every problem in this book, then you will learn Discrete Mathematics
    - You learned Discrete Mathematics
    - Therefore you did every problem in the book
  - Denying the hypothesis
    - p \rightarrow q, \neg p, therefore \neg q
Example (#1)

• Break up into groups of 3 or 4

• Determine whether this argument is valid:
  • Lynn works part time or full time.
  • If Lynn does not play on the team, then she does not work part time.
  • If Lynn plays on the team, she is busy.
  • Lynn does not work full time.
  • Therefore, Lynn is busy.
Examples (#1 Solution)

- Lynn works part time
- Lynn works full time
- Lynn plays on the team
- Lynn is busy,

1) \( p \lor f \) premise
2) \( \neg t \rightarrow \neg p \) premise
3) \( t \rightarrow b \) premise
4) \( \neg f \) premise
5) \( p \) Disjunctive syllogism 1 and 4
6) \( p \rightarrow t \) Contrapositive of 2
7) \( t \) Modus ponens on 5 and 6
8) \( b \) Modus ponens on 7 and 3

-Lynn works part time or full time.
-If Lynn does not play on the team, then she does not work part time.
-If Lynn plays on the team, she is busy.
-Lynn does not work full time.

Therefore, Lynn is busy.
More Examples (#2)

Prove or disprove that the square of every even integer ends in 0, 4, or 6.
More Examples (#2 solutions)

Prove or disprove that the square of every even integer ends in 0, 4, or 6.

Every even integer $n$ can be written as $n = 10k + r$ where $r = 0, 2, 4, 6, 8$. (For example, $34 = 10 \cdot 3 + 4$ and $6 = 6 \cdot 0 + 6$.)

Examine each of these five cases separately:

$n = 10k + 0$: $n^2 = 100k^2$, which ends in 0 (it is a multiple of 10)

$n = 10k + 2$: $n^2 = 100k^2 + 40k + 4 = 10(10k^2 + 4k) + 4$ and hence ends in 4

$n = 10k + 4$: $n^2 = 100k^2 + 80k + 16 = 10(10k^2 + 8k + 1) + 6$ and hence ends in 6

$n = 10k + 6$: $n^2 = 100k^2 + 120k + 36 = 10(10k^2 + 12k + 3) + 6$ and hence ends in 6

$n = 10k + 8$: $n^2 = 100k^2 + 160k + 64 = 10(10k^2 + 16k + 6) + 4$ and hence ends in 4.
More Examples (#3, #4, and #5)

In the questions below suppose the variable $x$ represents students and $y$ represents courses, and:

$F(x)$: $x$ is a freshman
$A(x)$: $x$ is a part-time student
$T(x,y)$: $x$ is taking $y$

Write the statement in good English without using variables in your answers.

3. $F$(Mikko).

4. $\neg \exists y \ T$(Joe,$y$).

5. $\exists x (A(x) \land \neg F(x))$. 
More Examples (#3,#4, and #5 Solutions)

In the questions below suppose the variable $x$ represents students and $y$ represents courses, and:

- $F(x)$: $x$ is a freshman
- $A(x)$: $x$ is a part-time student
- $T(x,y)$: $x$ is taking $y$

Write the statement in good English without using variables in your answers.

3. $F($Mikko$)$. Mikko is a freshman
4. $\neg \exists y \ T($Joe,$y$). Joe is not taking any course
5. $\exists x (A(x) \land \neg F(x))$. Some part-time students are not freshman
More Examples (#6 & #7)

• 6) Determine whether $p \rightarrow (q \rightarrow r)$ and $p \rightarrow (q \land r)$ are equivalent

• 7) Prove that $(\neg p \land (p \lor q)) \rightarrow q$ is a tautology using propositional equivalence and the laws of logic
More Examples (#6 solutions)

• 6) Determine whether \( p \rightarrow (q \rightarrow r) \)
and \( p \rightarrow (q \land r) \) are equivalent.

Ans: Not equivalent. Let \( q \) be false and \( p \) and \( r \) be true.
More Examples (#7 solutions)

Prove that \((\neg p \land (p \lor q)) \rightarrow q\) is a tautology using propositional equivalence and the laws of logic

Ans:

\[
(\neg p \land (p \lor q)) \rightarrow q \\
\equiv ((\neg p \land p) \lor (\neg p \land q)) \rightarrow q \quad \text{(Distributivity)} \\
\equiv (F \lor (\neg p \land q)) \rightarrow q \quad \text{(Uniqueness)} \\
\equiv (\neg p \land q) \rightarrow q \quad \text{(Domination)} \\
\equiv \neg (\neg p \land q) \lor q \quad (Defn \rightarrow) \\
\equiv (\neg p \lor \neg q) \lor q \quad \text{(De Morgan’s)} \\
\equiv (p \lor \neg q) \lor q \\
\equiv p \lor (\neg q \lor q) \quad \text{(Associativity)} \\
\equiv p \lor (T) \\
\equiv T
\]
• 8) Find a proposition with three variables $p$, $q$, and $r$ that is true when exactly one of the three variables is true, and false otherwise.

• 9) Find a proposition with three variables $p$, $q$, and $r$ that is never true.
More Examples (#8 & #9 solutions)

• 8) Find a proposition with three variables $p, q,$ and $r$ that is true when exactly one of the three variables is true, and false otherwise
   
   Ans: $(p \land \neg q \land \neg r) \lor (\neg p \land q \land \neg r) \lor (\neg p \land \neg q \land r)$

• 9) Find a proposition with three variables $p, q,$ and $r$ that is never true
   
   Ans: $(p \land \neg p) \lor (q \land \neg q) \lor (r \land \neg r)$
More Examples (#10)

10) Prove the following identities

- A) \( B \cup (\emptyset \cap A) = B \)

- B) \( \overline{A} \cap U = A \)

- C) \( (A \cap B) \cup (A \cap \overline{B}) = A \)

- D) \( (C \cup A) \cap (B \cup A) = (A \cup (B \cap C)) \)
More Examples (#10 Solutions)

• **10) Prove the following identities**

  - **A)** \( B \cup (\emptyset \cap A) = B \)

    \[
    B \cup (\emptyset \cap A) = B \cup \emptyset = B
    \]

  - **B)** \( (\overline{A} \cap U) = A \)

    \[
    (\overline{A} \cap U) = (\overline{A}) \cup \overline{U} = A \cup \overline{U} = A \cup \emptyset = A
    \]
More Examples (#10 Solutions)

• 10) Prove the following identities

\[- c) (A \cap B) \cup (A \cap \overline{B}) = A
\]
\[ (A \cap B) \cup (A \cap \overline{B}) = A \cap (B \cup \overline{B}) = A \cap U = A \]

\[- d) (C \cup A) \cap (B \cup A) = (A \cup (B \cap C))\]

• Just use distributivity
More Examples (#11)

11) Prove that

\[ 1 + 2 + \ldots + n = \frac{n(n + 1)}{2} \]
More Examples (#11 Solutions)

11) Prove that \( 1 + 2 + \ldots + n = \frac{n(n + 1)}{2} \)

- Base case: \( n = 1, \quad 1(2) / 2 = 1 \)
- IH: \( 1 + 2 + \ldots + n-1 = (n-1)n / 2 \)
- Inductive case:

\[
\sum_{i=1}^{n} i = \sum_{i=1}^{n-1} i + n = (n-1)n/2 + n - 1 = n(n+1)/2
\]