SECTION 10.2 Graph Terminology and Special Types of Graphs

2. In this pseudograph there are 5 vertices and 13 edges. The degree of vertex $a$ is 6, since in addition to the 4 nonloops incident to $a$, there is a loop contributing 2 to the degree. The degrees of the other vertices are $\deg(b) = 6$, $\deg(c) = 6$, $\deg(d) = 5$, and $\deg(e) = 3$. There are no pendant or isolated vertices in this pseudograph.

4. For the graph in Exercise 1, the sum is $2 + 4 + 1 + 0 + 2 + 3 = 12 = 2 \cdot 6$; there are 6 edges. For the pseudograph in Exercise 2, the sum is $6 + 6 + 6 + 5 + 3 = 26 = 2 \cdot 13$; there are 13 edges. For the pseudograph in Exercise 3, the sum is $3 + 2 + 4 + 0 + 6 + 0 + 4 + 2 + 3 = 24 = 2 \cdot 12$; there are 12 edges.

6. Model this problem by letting the vertices of a graph be the people at the party, with an edge between two people if they shake hands. Then the degree of each vertex is the number of people the person that vertex represents shakes hands with. By Theorem 1 the sum of the degrees is even (it is $2e$).

8. In this directed multigraph there are 4 vertices and 8 edges. The degrees are $\deg^- (a) = 2$, $\deg^+ (a) = 2$, $\deg^- (b) = 3$, $\deg^+ (b) = 4$, $\deg^- (c) = 2$, $\deg^+ (c) = 1$, $\deg^- (d) = 1$, and $\deg^+ (d) = 1$.

10. For Exercise 7 the sum of the in-degrees is $3 + 1 + 2 + 1 = 7$, and the sum of the out-degrees is $1 + 2 + 1 + 3 = 7$; there are 7 edges. For Exercise 8 the sum of the in-degrees is $2 + 3 + 2 + 1 = 8$, and the sum of the out-degrees is $2 + 4 + 1 + 1 = 8$; there are 8 edges. For Exercise 9 the sum of the in-degrees is $6 + 1 + 2 + 4 + 0 = 13$, and the sum of the out-degrees is $1 + 5 + 5 + 2 + 0 = 13$; there are 13 edges.

12. Since there is an edge from a person to each of his or her acquaintances, the degree of $v$ is the number of people $v$ knows. An isolated vertex would be a person who knows no one, and a pendant vertex would be a person who knows just one other person (it is doubtful that there are many, if any, isolated or pendant vertices). If the average degree is 1000, then the average person knows 1000 other people.

14. Since there is an edge from a person to each of the other actors with whom that person has appeared in a movie, the degree of $v$ is the number of other actors with whom that person has appeared. The neighborhood of $v$ is the set of actors with whom $v$ as appeared. An isolated vertex would be a person who has appeared only in movies in which he or she was the only actor, and a pendant vertex would be a person who has appeared with only one other actor in any movie (it is doubtful that there are many, if any, isolated or pendant vertices).

16. Since there is an edge from a page to each page that it links to, the outdegree of a vertex is the number of links on that page, and the in-degree of a vertex is the number of other pages that have a link to it.

18. This is essentially the same as Exercise 40 in Section 6.2, where the graph models the “know each other” relation on the people at the party. See the solution given for that exercise. The number of people a person knows is the degree of the corresponding vertex in the graph.

20. a) This graph has 7 vertices, with an edge joining each pair of distinct vertices.