M(x)) if the domain is all people. Note that “no one” became “for all . . . not.” If we let O(x, y) mean that person x owns item y, then we can rewrite this last one as \( \forall x (Y(x) \rightarrow \neg(O(x, \text{bicycle}) \land O(x, \text{motorcycle}))) \).

d) If we let \( H(x) \) be “x is happy,” then we have \( \exists x \neg H(x) \) if the domain is just your schoolmates, or \( \exists x (Y(x) \land \neg H(x)) \) if the domain is all people. If we let \( E(x, y) \) mean that person x is in mental state y, then we can rewrite this last one as \( \exists x (Y(x) \land \neg E(x, \text{happy})) \).

e) If we let \( T(x) \) be “x was born in the twentieth century,” then we have \( \forall x T(x) \) if the domain is just your schoolmates, or \( \forall x (Y(x) \rightarrow T(x)) \) if the domain is all people. If we let \( B(x, y) \) mean that person x was born in the \( y^{th} \) century, then we can rewrite this last one as \( \forall x (Y(x) \rightarrow B(x, 20)) \).

28. Let \( R(x) \) be “x is in the correct place”; let \( E(x) \) be “x is in excellent condition”; let \( T(x) \) be “x is a [or your] tool”; and let the domain of discourse be all things.

a) There exists something not in the correct place: \( \exists x \neg R(x) \).

b) If something is a tool, then it is in the correct place place and in excellent condition: \( \forall x (T(x) \rightarrow (R(x) \land E(x))) \).

c) \( \forall x (R(x) \land E(x)) \)

d) This is saying that everything fails to satisfy the condition: \( \forall x \neg(R(x) \land E(x)) \).

e) There exists a tool with this property: \( \exists x (T(x) \land \neg R(x) \land E(x)) \).

30. a) \( P(1, 3) \lor P(2, 3) \lor P(3, 3) \)  
b) \( P(1, 1) \land P(1, 2) \land P(1, 3) \)

c) \( \neg P(2, 1) \lor \neg P(2, 2) \lor \neg P(2, 3) \)  
d) \( \neg P(1, 2) \land \neg P(2, 2) \land \neg P(3, 2) \)

32. In each case we need to specify some propositional functions (predicates) and identify the domain of discourse.

a) Let \( F(x) \) be “x has fleas,” and let the domain of discourse be dogs. Our original statement is \( \forall x F(x) \). Its negation is \( \exists x \neg F(x) \). In English this reads “There is a dog that does not have fleas.”

b) Let \( H(x) \) be “x can add,” where the domain of discourse is horses. Then our original statement is \( \exists x H(x) \). Its negation is \( \forall x \neg H(x) \). In English this is rendered most simply as “No horse can add.”

c) Let \( C(x) \) be “x can climb,” and let the domain of discourse be koalas. Our original statement is \( \forall x C(x) \). Its negation is \( \exists x \neg C(x) \). In English this reads “There is a koala that cannot climb.”

d) Let \( F(x) \) be “x can speak French,” and let the domain of discourse be monkeys. Our original statement is \( \neg \exists x F(x) \) or \( \forall x \neg F(x) \). Its negation is \( \exists x F(x) \). In English this reads “There is a monkey that can speak French.”

e) Let \( S(x) \) be “x can swim” and let \( C(x) \) be “x can catch fish,” where the domain of discourse is pigs. Then our original statement is \( \exists x (S(x) \land C(x)) \). Its negation is \( \forall x \neg(S(x) \land C(x)) \), which could also be written \( \forall x (\neg S(x) \lor \neg C(x)) \) by De Morgan’s law. In English this is “No pig can both swim and catch fish,” or “Every pig either is unable to swim or is unable to catch fish.”

34. a) Let \( S(x) \) be “x obeys the speed limit,” where the domain of discourse is drivers. The original statement is \( \exists x \neg S(x) \), the negation is \( \forall x S(x) \), “All drivers obey the speed limit.”

b) Let \( S(x) \) be “x is serious,” where the domain of discourse is Swedish movies. The original statement is \( \forall x S(x) \), the negation is \( \exists x \neg S(x) \), “Some Swedish movies are not serious.”

c) Let \( S(x) \) be “x can keep a secret,” where the domain of discourse is people. The original statement is \( \neg \exists x S(x) \), the negation is \( \exists x S(x) \), “Some people can keep a secret.”

d) Let \( A(x) \) be “x has a good attitude,” where the domain of discourse is people in this class. The original statement is \( \exists x \neg A(x) \), the negation is \( \forall x A(x) \), “Everyone in this class has a good attitude.”

36. a) Since \( 1^2 = 1 \), this statement is false; \( x = 1 \) is a counterexample. So is \( x = 0 \) (these are the only two counterexamples).