In order to prove this for all integers \( n \geq 0 \), we first prove the basis step \( P(0) \) and then prove the inductive step, that \( P(k) \) implies \( P(k + 1) \). Now in \( P(0) \), the left-hand side has just one term, namely 2, and the right-hand side is \( (1 - (-7)^1)/4 = 8/4 = 2 \). Since \( 2 = 2 \), we have verified that \( P(0) \) is true. For the inductive step, we assume that \( P(k) \) is true (i.e., the displayed equation above), and derive from it the truth of \( P(k+1) \), which is the equation

\[
2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2 \cdot (-7)^k + 2 \cdot (-7)^{k+1} = \frac{1 - (-7)(k+1)+1}{4}.
\]

To prove an equation like this, it is usually best to start with the more complicated side and manipulate it until we arrive at the other side. In this case we start on the left. Note that all but the last term constitute precisely the left-hand side of \( P(k) \), and therefore by the inductive hypothesis, we can replace it by the right-hand side of \( P(k) \). The rest is algebra:

\[
[2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2 \cdot (-7)^k] + 2 \cdot (-7)^{k+1} = \frac{1 - (-7)^{k+1} + 2 \cdot (-7)^{k+1}}{4} = \frac{1 - (-7)^{k+1} + 8 \cdot (-7)^{k+1}}{4} = \frac{1 + 7 \cdot (-7)^{k+1}}{4} = \frac{1 - (-7) \cdot (-7)^{k+1}}{4} = \frac{1 - (-7)(k+1)+1}{4}.
\]

10. a) By computing the first few sums and getting the answers 1/2, 2/3, and 3/4, we guess that the sum is \( n/(n+1) \).

b) We prove this by induction. It is clear for \( n = 1 \), since there is just one term, 1/2. Suppose that

\[
\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}.
\]

We want to show that

\[
\left[ \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{k(k+1)} \right] + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}.
\]

Starting from the left, we replace the quantity in brackets by \( k/(k+1) \) (by the inductive hypothesis), and then do the algebra

\[
\frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k^2 + 2k + 1}{(k+1)(k+2)} = \frac{k+1}{k+2},
\]

yielding the desired expression.

12. We proceed by mathematical induction. The basis step \( (n = 0) \) is the statement that \((−1/2)^0 = (2+1)/(3\cdot1)\), which is the true statement that \( 1 = 1 \). Assume the inductive hypothesis, that

\[
\sum_{j=0}^{k} \left( -\frac{1}{2} \right)^j = \frac{2^{k+1} + (-1)^k}{3 \cdot 2^k}.
\]

We want to prove that

\[
\sum_{j=0}^{k+1} \left( -\frac{1}{2} \right)^j = \frac{2^{k+2} + (-1)^{k+1}}{3 \cdot 2^{k+1}}.
\]