we must show that
\[(A_1 \cap A_2 \cap \cdots \cap A_n \cap A_{n+1}) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \cdots \cap (A_n \cup B) \cap (A_{n+1} \cup B).\]

We have
\[
(A_1 \cap A_2 \cap \cdots \cap A_n \cap A_{n+1}) \cup B = ((A_1 \cap A_2 \cap \cdots \cap A_n) \cap A_{n+1}) \cup B
= ((A_1 \cap A_2 \cap \cdots \cap A_n) \cup B) \cap (A_{n+1} \cup B)
= (A_1 \cup B) \cap (A_2 \cup B) \cap \cdots \cap (A_n \cup B) \cap (A_{n+1} \cup B).
\]
The second line follows from the distributive law, and the third line follows from the inductive hypothesis.

42. If \(n = 1\) there is nothing to prove, and the \(n = 2\) case says that \((A_1 \cap \overline{B}) \cap (A_2 \cap \overline{B}) = (A_1 \cap A_2) \cap \overline{B}\), which is certainly true, since an element is in each side if and only if it is in all three of the sets \(A_1\), \(A_2\), and \(\overline{B}\). Those take care of the basis step. For the inductive step, assume that
\[(A_1 - B) \cap (A_2 - B) \cap \cdots \cap (A_n - B) = (A_1 \cap A_2 \cap \cdots \cap A_n) - B;\]
we must show that
\[(A_1 - B) \cap (A_2 - B) \cap \cdots \cap (A_n - B) \cap (A_{n+1} - B) = (A_1 \cap A_2 \cap \cdots \cap A_n \cap A_{n+1}) - B.\]
We have
\[
(A_1 - B) \cap (A_2 - B) \cap \cdots \cap (A_n - B) \cap (A_{n+1} - B)
= ((A_1 - B) \cap (A_2 - B) \cap \cdots \cap (A_n - B)) \cap (A_{n+1} - B)
= ((A_1 \cap A_2 \cap \cdots \cap A_n) - B) \cap (A_{n+1} - B)
= (A_1 \cap A_2 \cap \cdots \cap A_n \cap A_{n+1}) - B.
\]
The third line follows from the inductive hypothesis, and the fourth line follows from the \(n = 2\) case.

44. If \(n = 1\) there is nothing to prove, and the \(n = 2\) case says that \((A_1 \cap \overline{B}) \cup (A_2 \cap \overline{B}) = (A_1 \cup A_2) \cap \overline{B}\), which is the distributive law (see Table 1 in Section 2.2). Those take care of the basis step. For the inductive step, assume that
\[(A_1 - B) \cup (A_2 - B) \cup \cdots \cup (A_n - B) = (A_1 \cup A_2 \cup \cdots \cup A_n) - B;\]
we must show that
\[(A_1 - B) \cup (A_2 - B) \cup \cdots \cup (A_n - B) \cup (A_{n+1} - B) = (A_1 \cup A_2 \cup \cdots \cup A_n \cup A_{n+1}) - B.\]
We have
\[
(A_1 - B) \cup (A_2 - B) \cup \cdots \cup (A_n - B) \cup (A_{n+1} - B)
= ((A_1 - B) \cup (A_2 - B) \cup \cdots \cup (A_n - B)) \cup (A_{n+1} - B)
= ((A_1 \cup A_2 \cup \cdots \cup A_n) - B) \cup (A_{n+1} - B)
= (A_1 \cup A_2 \cup \cdots \cup A_n \cup A_{n+1}) - B.
\]
The third line follows from the inductive hypothesis, and the fourth line follows from the \(n = 2\) case.

46. This proof will be similar to the proof in Example 10. The basis step is clear, since for \(n = 3\), the set has exactly one subset containing exactly three elements, and \(3(3-1)(3-2)/6 = 1\). Assume the inductive hypothesis, that a set with \(n\) elements has \(n(n-1)(n-2)/6\) subsets with exactly three elements; we want to prove that a set \(S\) with \(n+1\) elements has \((n+1)n(n-1)/6\) subsets with exactly three elements. Fix an element \(a\) in \(S\), and let \(T\) be the set of elements of \(S\) other than \(a\). There are two varieties of subsets of \(S\) containing exactly three elements. First there are those that do not contain \(a\). These are precisely the three-element subsets of \(T\), and by the inductive hypothesis, there are \(n(n-1)(n-2)/6\) of them. Second, there are those that contain \(a\) together with two elements of \(T\). Therefore there are just as many of these subsets as there are two-element subsets of \(T\). By Exercise 45, there are exactly \(n(n-1)/2\) such subsets of \(T\); therefore there are also \(n(n-1)/2\) three-element subsets of \(S\) containing \(a\). Thus the total number of subsets of \(S\) containing exactly three elements is \((n(n-1)(n-2)/6) + n(n-1)/2\), which simplifies algebraically to \((n+1)n(n-1)/6\), as desired.