by 3. The remaining 6000 are not.

e) For this and the next three parts we need to note first that one fifth of the numbers in this range, or 1800 of them, are divisible by 5, and one seventh of them, or 1286 are divisible by 7. [This last calculation is a little more subtle than we let on, since 9000 is not divisible by 7 (the quotient is 1285.71...). But 1001 is divisible by 7, and 1001 + 1285 \cdot 7 = 9996, so there are indeed 1286, and not 1285 such multiples. (By contrast, in the range 1002 to 10001, inclusive, which also includes 9000 numbers, there are only 1285 multiples of 7.) We also need to know how many of these numbers are divisible by both 5 and 7, which means divisible by 35.
The answer, by the similar reasoning, is 257, namely those multiples from 29 \cdot 35 = 1015 to 285 \cdot 35 = 9975.
(One more note: We could also have come up with these numbers more formally, using the ideas in Section 8.5, especially Example 2. We could find the number of multiples less than 10,000 and subtract the number of multiples less than 1000.) Now to the problem at hand. The number of numbers divisible by 5 or 7 is the number of numbers divisible by 5, plus the number of numbers divisible by 7, minus (because of having overcounted) the number of numbers divisible by both. So our answer is 1800 + 1286 – 257 = 2829.
f) Since we just found that 2829 of these numbers are divisible by either 5 or 7, it follows that the rest of them, 9000 – 2829 = 6171, are not.
g) We noted in the solution to part (e) that 1800 numbers are divisible by 5, and 257 of these are also divisible by 7. Therefore 1800 – 257 = 1543 numbers in our range are divisible by 5 but not by 7.
h) We found this as part of our solution to part (e), namely 257.

26. a) There are 10 ways to choose the first digit, 9 ways to choose the second, and so on; therefore the answer is 10 \cdot 9 \cdot 8 \cdot 7 = 5040.
b) There are 10 ways to choose each of the first three digits and 5 ways to choose the last; therefore the answer is 10^3 \cdot 5 = 5000.
c) There are 4 ways to choose the position that is to be different from 9, and 9 ways to choose the digit to go there. Therefore there are 4 \cdot 9 = 36 such strings.

28. 10^326^3 + 26^310^3 = 35,152,000

30. 26^310^3 + 26^410^2 = 63,273,600

32. a) By the product rule, the answer is 26^8 = 208,827,064,576.
b) By the product rule, the answer is 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 = 62,990,928,000.
c) This is the same as part (a), except that there are only seven slots to fill, so the answer is 26^7 = 8,031,810,176.
d) This is similar to (b), except that there is only one choice in the first slot, rather than 26, so the answer is 1 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 = 2,422,728,000.
e) This is the same as part (c), except that there are only six slots to fill, so the answer is 26^6 = 308,915,776.
f) This is the same as part (e); again there are six slots to fill, so the answer is 26^6 = 308,915,776.
g) This is the same as part (f), except that there are only four slots to fill, so the answer is 26^4 = 456,976.
We are assuming that the question means that the legal strings are BO???BO, where any letters can fill the middle four slots.
h) By part (f), there are 26^4 strings that start with the letters BO in that order. By the same argument, there are 26^6 strings that end that way. By part (g), there are 26^4 strings that both start and end with the letters BO in that order. Therefore by the inclusion–exclusion principle, the answer is 26^6 + 26^6 – 26^4 = 617,374,576.

34. In each case the answer is n^{10}, where n is the number of elements in the codomain, since there are n choices for a function value for each of the 10 elements in the domain.
a) 2^{10} = 1024    b) 3^{10} = 59,049    c) 4^{10} = 1,048,576    d) 5^{10} = 9,765,625