CHAPTER 7
Discrete Probability

SECTION 7.1  An Introduction to Discrete Probability

2. The probability is $1/6 \approx 0.17$, since there are six equally likely outcomes.

4. Since April has 30 days, the answer is $30/366 = 5/61 \approx 0.082$.

6. There are 16 cards that qualify as being an ace or a heart, so the answer is $16/52 = 4/13 \approx 0.31$. We could also compute this from Theorem 2 as $4/52 + 13/52 - 1/52$.

8. We saw in Example 11 of Section 6.3 that there are $C(52, 5)$ possible poker hands, and we assume by symmetry that they are all equally likely. In order to solve this problem, we need to compute the number of poker hands that contain the ace of hearts. There is no choice about choosing the ace of hearts. To form the rest of the hand, we need to choose 4 cards from the 51 remaining cards, so there are $C(51, 4)$ hands containing the ace of hearts. Therefore the answer to the question is the ratio

$$\frac{C(51, 4)}{C(52, 5)} = \frac{5}{52} \approx 9.6\%.$$

The problem can also be done by subtracting from 1 the answer to Exercise 9, since a hand contains the ace of hearts if and only if it is not the case that it does not contain the ace of hearts.

10. This is similar to Exercise 8. We need to compute the number of poker hands that contain the two of diamonds and the three of spades. There is no choice about choosing these two cards. To form the rest of the hand, we need to choose 3 cards from the 50 remaining cards, so there are $C(50, 3)$ hands containing these two specific cards. Therefore the answer to the question is the ratio

$$\frac{C(50, 3)}{C(52, 5)} = \frac{5}{663} \approx 0.0075.$$

12. There are 4 ways to specify the ace. Once the ace is chosen for the hand, there are $C(48, 4)$ ways to choose nonaces for the remaining four cards. Therefore there are $4C(48, 4)$ hands with exactly one ace. Since there are $C(52, 5)$ equally likely hands, the answer is the ratio

$$\frac{4C(48, 4)}{C(52, 5)} \approx 0.30.$$

14. We saw in Example 11 of Section 6.3 that there are $C(52, 5) = 2,598,960$ different hands, and we assume by symmetry that they are all equally likely. We need to count the number of hands that have 5 different kinds (ranks). There are $C(13, 5)$ ways to choose the kinds. For each card, there are then 4 ways to choose the suit. Therefore there are $C(13, 5) \cdot 4^5 = 1,317,888$ ways to choose the hand. Thus the probability is $1317888/2598960 = 2112/4165 \approx 0.51$. 