6. We can exploit symmetry in answering these.
   a) Since 1 has either to precede 3 or to follow it, and there is no reason that one of these should be any more likely than the other, we immediately see that the answer is 1/2. We could also simply list all 6 permutations and count that 3 of them have 1 preceding 3, namely 123, 132, and 213.
   b) By the same reasoning as in part (a), the answer is again 1/2.
   c) The stated conditions force 3 to come first, so only 312 and 321 are allowed. Therefore the answer is 2/6 = 1/3.

8. We exploit symmetry in answering many of these.
   a) Since 1 has either to precede 2 or to follow it, and there is no reason that one of these should be any more likely than the other, we immediately see that the answer is 1/2.
   b) By the same reasoning as in part (a), the answer is again 1/2.
   c) For 1 immediately to precede 2, we can think of these two numbers as glued together in forming the permutation. Then we are really permuting $n - 1$ numbers—the single numbers from 3 through $n$ and the one glued object, 12. There are $(n - 1)!$ ways to do this. Since there are $n!$ permutations in all, the probability of randomly selecting one of these is $(n - 1)!/n! = 1/n$.
   d) Half of the permutations have 1 preceding 1. Of these permutations, half of them have $n - 1$ preceding 2. Therefore one fourth of the permutations satisfy these conditions, so the probability is 1/4.
   e) Looking at the relative placements of 1, 2, and $n$, we see that one third of the time, $n$ will come first. Therefore the answer is 1/3.

10. Note that there are 26! permutations of the letters, so the denominator in all of our answers is 26!. To find the numerator, we have to count the number of ways that the given event can happen. Alternatively, in some cases we may be able to exploit symmetry.
   a) There are 13! possible arrangements of the first 13 letters of the permutation, and in only one of these are they in alphabetical order. Therefore the answer is 1/13!.
   b) Once these two conditions are met, there are 24! ways to choose the remaining letters for positions 2 through 25. Therefore the answer is 24!/26! = 1/650.
   c) In effect we are forming a permutation of 25 items—the letters b through y and the double letter combination ax or xa. There are 25! ways to permute these items, and for each of these permutations there are two choices as to whether a or z comes first. Thus there are 2 · 25! ways for form such a permutation, and therefore the answer is 2 · 25!/26! = 1/13.
   d) By part (c), the probability that a and b are next to each other is 1/13. Therefore the probability that a and b are not next to each other is 12/13.
   e) There are six ways this can happen: $ax^{24}z$, $zx^{24}a$, $axz^{23}a$, $ax^{23}z$, $xz^{23}a$, and $zx^{23}ax$, where $x$ stands for any letter other than a and z (but of course all the $x$’s are different in each permutation). In each of these there are 24! ways to permute the letters other than a and z, so there are 24! permutations of each type. This gives a total of 6 · 24! permutations meeting the conditions, so the answer is $(6 · 24!)/26! = 3/325$.
   f) Looking at the relative placements of z, a, and b, we see that one third of the time, z will come first. Therefore the answer is 1/3.

12. Clearly $p(E \cup F) \geq p(E) = 0.8$. Also, $p(E \cup F) \leq 1$. If we apply Theorem 2 from Section 7.1, we can rewrite this as $p(E) + p(F) − p(E \cap F) \leq 1$, or $0.8 + 0.6 − p(E \cap F) \leq 1$. Solving for $p(E \cap F)$ gives $p(E \cap F) \geq 0.4$.

14. The basis step $n = 1$ is the trivial statement that $p(E_1) \geq p(E_1)$, and the case $n = 2$ was done in Exercise 13. Assume the inductive hypothesis:
   
   $p(E_1 \cap E_2 \cap \cdots \cap E_n) \geq p(E_1) + p(E_2) + \cdots + p(E_n) − (n − 1)$