b) This is satisfiable by, for example, setting \( p \) to be false (that takes care of the first, second, and fourth disjunctions), \( s \) to be false (for the third and sixth disjunctions), \( q \) to be true (for the fifth disjunction), and \( r \) to be anything.

c) It is not hard to find a satisfying truth assignment, such as \( p, q, \) and \( s \) true, and \( r \) false.

64. Recall that \( p(i,j,n) \) asserts that the cell in row \( i \), column \( j \) contains the number \( n \). Thus \( \bigvee_{n=1}^{9} p(i,j,n) \) asserts that this cell contains at least one number. To assert that every cell contains at least one number, we take the conjunction of these statements over all cells: \( \bigwedge_{i=1}^{9} \bigwedge_{j=1}^{9} \bigvee_{n=1}^{9} p(i,j,n) \).

66. There are nine blocks, in three rows and three columns. Let \( r \) and \( s \) index the row and column of the block, respectively, where we start counting at 0, so that \( 0 \leq r \leq 2 \) and \( 0 \leq s \leq 2 \). (For example, \( r = 0, s = 1 \) corresponds to the block in the first row of blocks and second column of blocks.) The key point is to notice that the block corresponding to the pair \( (r,s) \) contains the cells that are in rows \( 3r + 1 \), \( 3r + 2 \), and \( 3r + 3 \) and columns \( 3s + 1 \), \( 3s + 2 \), and \( 3s + 3 \). Therefore \( p(3r + i, 3s + j,n) \) asserts that a particular cell in this block contains the number \( n \), where \( 1 \leq i \leq 3 \) and \( 1 \leq j \leq 3 \). If we take the disjunction over all these values of \( i \) and \( j \), then we obtain \( \bigvee_{i=1}^{3} \bigvee_{j=1}^{3} p(3r + i, 3s + j,n) \), asserting that some cell in this block contains the number \( n \). Because we want this to be true for every number and for every block, we form the triply-indexed conjunction given in the text.

SECTION 1.4 Predicates and Quantifiers

2. a) This is true, since there is an \( a \) in orange.  
   b) This is false, since there is no \( a \) in lemon.  
   c) This is false, since there is no \( a \) in true.  
   d) This is true, since there is an \( a \) in false.

4. a) Here \( x \) is still equal to 0, since the condition is false.  
   b) Here \( x \) is still equal to 1, since the condition is false.  
   c) This time \( x \) is equal to 1 at the end, since the condition is true, so the statement \( x := 1 \) is executed.

6. The answers given here are not unique, but care must be taken not to confuse nonequivalent sentences. Parts (c) and (f) are equivalent; and parts (d) and (e) are equivalent. But these two pairs are not equivalent to each other.

   a) Some student in the school has visited North Dakota. (Alternatively, there exists a student in the school who has visited North Dakota.)

   b) Every student in the school has visited North Dakota. (Alternatively, all students in the school have visited North Dakota.)

   c) This is the negation of part (a): No student in the school has visited North Dakota. (Alternatively, there does not exist a student in the school who has visited North Dakota.)

   d) Some student in the school has not visited North Dakota. (Alternatively, there exists a student in the school who has not visited North Dakota.)

   e) This is the negation of part (b): It is not true that every student in the school has visited North Dakota. (Alternatively, not all students in the school have visited North Dakota.)

   f) All students in the school have not visited North Dakota. (This is technically the correct answer, although common English usage takes this sentence to mean—incorrectly—the answer to part (e). To be perfectly clear, one could say that every student in this school has failed to visit North Dakota, or simply that no student has visited North Dakota.)