12. This is true. Suppose that \( a/b \) is a nonzero rational number and that \( x \) is an irrational number. We must prove that the product \( xa/b \) is also irrational. We give a proof by contradiction. Suppose that \( xa/b \) were rational. Since \( a/b \neq 0 \), we know that \( a \neq 0 \), so \( b/a \) is also a rational number. Let us multiply this rational number \( b/a \) by the assumed rational number \( xa/b \). By Exercise 26, the product is rational. But the product is \( (b/a)(xa/b) = x \), which is irrational by hypothesis. This is a contradiction, so in fact \( xa/b \) must be irrational, as desired.

14. If \( x \) is rational and not zero, then by definition we can write \( x = p/q \), where \( p \) and \( q \) are nonzero integers. Since \( 1/x \) is then \( q/p \) and \( p \neq 0 \), we can conclude that \( 1/x \) is rational.

16. We give a proof by contraposition. If it is not true than \( m \) is even or \( n \) is even, then \( m \) and \( n \) are both odd. By Exercise 6, this tells us that \( mn \) is odd, and our proof is complete.

18. a) We must prove the contrapositive: If \( n \) is odd, then \( 3n + 2 \) is odd. Assume that \( n \) is odd. Then we can write \( n = 2k + 1 \) for some integer \( k \). Then \( 3n + 2 = 3(2k + 1) + 2 = 6k + 5 = 2(3k + 2) + 1 \). Thus \( 3n + 2 \) is two times some integer plus 1, so it is odd.

b) Suppose that \( 3n + 2 \) is even and that \( n \) is odd. Since \( 3n + 2 \) is even, so is \( 3n \). If we add subtract an odd number from an even number, we get an odd number, so \( 3n - n = 2n \) is odd. But this is obviously not true. Therefore our supposition was wrong, and the proof by contradiction is complete.

20. We need to prove the proposition “If 1 is a positive integer, then \( 1^2 \geq 1 \).” The conclusion is the true statement \( 1 \geq 1 \). Therefore the conditional statement is true. This is an example of a trivial proof, since we merely showed that the conclusion was true.

22. We give a proof by contradiction. Suppose that we don’t get a pair of blue socks or a pair of black socks. Then we drew at most one of each color. This accounts for only two socks. But we are drawing three socks. Therefore our supposition that we did not get a pair of blue socks or a pair of black socks is incorrect, and our proof is complete.

24. We give a proof by contradiction. If there were at most two days falling in the same month, then we could have at most \( 2 \cdot 12 = 24 \) days, since there are 12 months. Since we have chosen 25 days, at least three of them must fall in the same month.

26. We need to prove two things, since this is an “if and only if” statement. First let us prove directly that if \( n \) is even then \( 7n + 4 \) is even. Since \( n \) is even, it can be written as \( 2k \) for some integer \( k \). Then \( 7n + 4 = 14k + 4 = 2(7k + 2) \). This is \( 2 \) times an integer, so it is even, as desired. Next we give a proof by contraposition that if \( 7n + 4 \) is even then \( n \) is even. So suppose that \( n \) is not even, i.e., that \( n \) is odd. Then \( n \) can be written as \( 2k + 1 \) for some integer \( k \). Thus \( 7n + 4 = 14k + 11 = 2(7k + 5) + 1 \). This is \( 1 \) more than \( 2 \) times an integer, so it is odd. That completes the proof by contraposition.

28. There are two things to prove. For the “if” part, there are two cases. If \( m = n \), then of course \( m^2 = n^2 \); if \( m = -n \), then \( m^2 = (-n)^2 = (-1)^2n^2 = n^2 \). For the “only if” part, we suppose that \( m^2 = n^2 \). Putting everything on the left and factoring, we have \( (m + n)(m - n) = 0 \). Now the only way that a product of two numbers can be zero is if one of them is zero. Therefore we conclude that either \( m + n = 0 \) (in which case \( m = -n \)), or else \( m - n = 0 \) (in which case \( m = n \)), and our proof is complete.

30. We write these in symbols: \( a < b \), \( (a + b)/2 > a \), and \( (a + b)/2 < b \). The latter two are equivalent to \( a + b > 2a \) and \( a + b < 2b \), respectively, and these are in turn equivalent to \( b > a \) and \( a < b \), respectively. It is now clear that all three statements are equivalent.