SECTION 2.2  Set Operations

2. a) \( A \cap B \)  
   b) \( A \cap B \), which is the same as \( A - B \)  
   c) \( A \cup B \)  
   d) \( A \cup B \)

4. Note that \( A \subseteq B \).
   a) \( \{a, b, c, d, e, f, g, h\} = B \)  
   b) \( \{a, b, c, d, e\} = A \)  
   c) There are no elements in \( A \) that are not in \( B \), so the answer is \( \emptyset \).  
   d) \( \{f, g, h\} \)

6. a) \( A \cup \emptyset = \{x \mid x \in A \lor x \in \emptyset\} = \{x \mid x \in A \lor F\} = \{x \mid x \in A\} = A \)  
   b) \( A \cup U = \{x \mid x \in A \land x \in U\} = \{x \mid x \in A \land T\} = \{x \mid x \in A\} = A \)

8. a) \( A \cap A = \{x \mid x \in A \land x \in A\} = \{x \mid x \in A\} = A \)  
   b) \( A \cap A = \{x \mid x \in A \land x \in A\} = \{x \mid x \in A\} = A \)

10. a) \( A - \emptyset = \{x \mid x \in A \land x \notin \emptyset\} = \{x \mid x \in A \land T\} = \{x \mid x \in A\} = A \)  
    b) \( \emptyset - A = \{x \mid x \in \emptyset \land x \notin A\} = \{x \mid F \land x \notin A\} = \{x \mid F\} = \emptyset \)

12. We will show that these two sets are equal by showing that each is a subset of the other. Suppose \( x \in A \cup (A \cap B) \). Then \( x \in A \) or \( x \in A \cap B \) by the definition of union. In the former case, we have \( x \in A \), and in the latter case we have \( x \in A \) and \( x \in B \) by the definition of intersection; thus in any event, \( x \in A \), so we have proved that the left-hand side is a subset of the right-hand side. Conversely, let \( x \in A \). Then by the definition of union, \( x \in A \cup (A \cap B) \) as well. Thus we have shown that the right-hand side is a subset of the left-hand side.

14. Since \( A = (A - B) \cup (A \cap B) \), we conclude that \( A = \{1, 5, 7, 8\} \cup \{3, 6, 9\} = \{1, 3, 5, 6, 7, 8, 9\} \). Similarly \( B = (B - A) \cup (A \cap B) = \{2, 10\} \cup \{3, 6, 9\} = \{2, 3, 6, 9, 10\} \).

16. a) If \( x \) is in \( A \cap B \), then perforce it is in \( A \) (by definition of intersection).  
    b) If \( x \) is in \( A \), then perforce it is in \( A \cup B \) (by definition of union).  
    c) If \( x \) is in \( A - B \), then perforce it is in \( A \) (by definition of difference).  
    d) If \( x \in A \) then \( x \notin B - A \). Therefore there can be no elements in \( A \cap (B - A) \), so \( A \cap (B - A) = \emptyset \).  
    e) The left-hand side consists precisely of those things that are either elements of \( A \) or else elements of \( B \) but not \( A \), in other words, things that are elements of either \( A \) or \( B \) (or, of course, both). This is precisely the definition of the right-hand side.

18. a) Suppose that \( x \in A \cup B \). Then either \( x \in A \) or \( x \in B \). In either case, certainly \( x \in A \cup B \cup C \). This establishes the desired inclusion.  
    b) Suppose that \( x \in A \cap B \cap C \). Then \( x \) is in all three of these sets. In particular, it is in both \( A \) and \( B \) and therefore in \( A \cap B \), as desired.  
    c) Suppose that \( x \in (A - B) - C \). Then \( x \) is in \( A - B \) but not in \( C \). Since \( x \in A - B \), we know that \( x \in A \) (we also know that \( x \notin B \), but that won’t be used here). Since we have established that \( x \in A \) but \( x \notin C \), we have proved that \( x \in A - C \).  
    d) To show that the set given on the left-hand side is empty, it suffices to assume that \( x \) is some element in that set and derive a contradiction, thereby showing that no such \( x \) exists. So suppose that \( x \in (A - C) \cap (C - B) \). Then \( x \in A - C \) and \( x \in C - B \). The first of these statements implies by definition that \( x \notin C \), while the second implies that \( x \in C \). This is impossible, so our proof by contradiction is complete.  
    e) To establish the equality, we need to prove inclusion in both directions. To prove that \( (B - A) \cup (C - A) \subseteq (B \cup C) - A \), suppose that \( x \in (B - A) \cup (C - A) \). Then either \( x \in (B - A) \) or \( x \in (C - A) \). Without loss of