14. a) This is clearly onto, since \( f(0, -n) = n \) for every integer \( n \).

b) This is not onto, since, for example, 2 is not in the range. To see this, if \( m^2 - n^2 = (m - n)(m + n) = 2 \), then \( m \) and \( n \) must have same parity (both even or both odd). In either case, both \( m - n \) and \( m + n \) are then even, so this expression is divisible by 4 and hence cannot equal 2.

c) This is clearly onto, since \( f(0, n - 1) = n \) for every integer \( n \).

d) This is onto. To achieve negative values we set \( m = 0 \), and to achieve nonnegative values we set \( n = 0 \).

e) This is not onto, for the same reason as in part (b). In fact, the range here is clearly a subset of the range in that part.

16. a) This would normally be one-to-one, unless somehow two students in the class had a strange mobile phone service in which they shared the same phone number.

b) This is surely one-to-one; otherwise the student identification number would not “identify” students very well!

c) This is almost surely not one-to-one; unless the class is very small, it is very likely that two students will receive the same grade.

d) This function will be one-to-one as long as no two students in the class hail from the same town (which is rather unlikely, so the function is probably not one-to-one).

18. Student answers may vary, depending on the choice of codomain.

a) A codomain could be all ten-digit positive integers; the function is not onto because there are many possible phone numbers assigned to people not in the class.

b) Under some student record systems, the student number consists of eight digits, so the codomain could be all natural numbers less than 100,000,000. The class does not have 100,000,000 students in it, so this function is not onto.

c) A codomain might be \( \{A, B, C, D, F\} \) (the answer depends on the grading system used at that school). If there were people at all five performance levels in this class, then the function would be onto. If not (for example, if no one failed the course), then it would not be onto.

d) The codomain could be the set of all cities and towns in the world. The function is clearly not onto. Alternatively, the codomain could be just the set of cities and towns from which the students in that class hail, in which case the function would be onto.

20. a) \( f(n) = n + 17 \)

b) \( f(n) = \lfloor n/2 \rfloor \)

c) We let \( f(n) = n - 1 \) for even values of \( n \), and \( f(n) = n + 1 \) for odd values of \( n \). Thus we have \( f(1) = 2 \), \( f(2) = 1 \), \( f(3) = 4 \), \( f(4) = 3 \), and so on. Note that this is just one function, even though its definition used two formulae, depending on the the parity of \( n \).

d) \( f(n) = 17 \)

22. If we can find an inverse, the function is a bijection. Otherwise we must explain why the function is not on-to-one or not onto.

a) This is a bijection since the inverse function is \( f^{-1}(x) = (4 - x)/3 \).

b) This is not one-to-one since \( f(17) = f(-17) \), for instance. It is also not onto, since the range is the interval \((-\infty, 7]\). For example, 42548 is not in the range.

c) This function is a bijection, but not from \( \mathbb{R} \) to \( \mathbb{R} \). To see that the domain and range are not \( \mathbb{R} \), note that \( x = -2 \) is not in the domain, and \( x = 1 \) is not in the range. On the other hand, \( f \) is a bijection from \( \mathbb{R} - \{-2\} \) to \( \mathbb{R} - \{1\} \), since its inverse is \( f^{-1}(x) = (1 - 2x)/(x - 1) \).

d) It is clear that this continuous function is increasing throughout its entire domain \( \mathbb{R} \) and it takes on both arbitrarily large values and arbitrarily small (large negative) ones. So it is a bijection. Its inverse is clearly \( f^{-1}(x) = \sqrt{x - 1} \).