Let \( w \) be any string. First notice that we can define a recursive definition of \( w^i \) as follows: 
\[ w^i = w w^{i-1} \text{ if } i \geq 1 \text{ and } w^i = \lambda \text{ if } i = 0. \]
Recall that \( \lambda \) denotes the empty string. This equality follows from an inductive proof. For the base case when \( i = 0 \), we return the empty string, which is 0 copies of \( w \). Inductively assume that \( w^i = w w^{i-1} \) for all \( 0 \leq i \leq k - 1 \) for some \( k \geq 1 \). Now we prove the case for \( i = k \). We see that \( w^i \) is \( i \) copies of \( w \). Remove the first copy, and we have \( i - 1 \) copies of \( w \) which is \( w^{i-1} \) by induction. Concatenate this with \( w \) and we have \( i \) copies of \( w \). Hence, \( w^i = w w^{i-1} \).

We use this to prove the original problem that \((w^R)^i = (w^i)^R\) by induction on \( i \). For \( i = 0 \), we have \((w^R)^0 = \lambda = \lambda^R = (w^i)^0\). Now assume that \((w^R)^i = (w^i)^R\) for all \( 0 \leq i \leq k \) for some \( k \geq 0 \). Consider the case where \( i = k + 1 \). We have that \((w^R)^{k+1} = w^R(w^R)^k\), by the above proof. Further, \( w^R(w^R)^k = w^R(w^k)^R\) by the inductive hypothesis. We also see that \( w^R(w^k)^R = (w^k w)^R\) by definition of reversal. Finally, \((w^k w)^R = (w^{k+1})^R\) again by the above proof.