there are 19 cables and 4 printers, the average number of computers per printer is 19/4, which is less than 5. Therefore some printer must be connected to fewer than 5 computers (the average of a set of numbers cannot be bigger than each of the numbers in the set). That means it is connected to 4 or fewer computers, so there are at least 4 computers that are not connected to it. If those 4 computers all needed a printer simultaneously, then they would be out of luck, since they are connected to at most the 3 other printers.

40. Let $K(x)$ be the number of other people at the party that person $x$ knows. The possible values for $K(x)$ are $0, 1, \ldots, n - 1$, where $n \geq 2$ is the number of people at the party. We cannot apply the pigeonhole principle directly, since there are $n$ pigeons and $n$ pigeonholes. However, it is impossible for both 0 and $n - 1$ to be in the range of $K$, since if one person knows everybody else, then nobody can know no one else (we assume that “knowing” is symmetric). Therefore the range of $K$ has at most $n - 1$ elements, whereas the domain has $n$ elements, so $K$ is not one-to-one, precisely what we wanted to prove.

42. a) The solution of Exercise 41, with 24 replaced by 2 and 149 replaced by 127, tells us that the statement is true.

b) The solution of Exercise 41, with 24 replaced by 23 and 149 replaced by 148, tells us that the statement is true.

c) We begin in a manner similar to the solution of Exercise 41. Look at $a_1, a_2, \ldots, a_{75}, a_1 + 25, \ldots, a_{75} + 25$, where $a_i$ is the total number of matches played up through and including hour $i$. Then $1 \leq a_1 < a_2 < \cdots < a_{75} \leq 125$, and $26 \leq a_1 + 25 < a_2 + 25 < \cdots < a_{75} + 25 \leq 150$. Now either these 150 numbers are precisely all the number from 1 to 150, or else by the pigeonhole principle we get, as in Exercise 41, $a_i = a_j + 25$ for some $i$ and $j$ and we are done. In the former case, however, since each of the numbers $a_i + 25$ is greater than or equal to 26, the numbers $1, 2, \ldots, 25$ must all appear among the $a_i$'s. But since the $a_i$'s are increasing, the only way this can happen is if $a_1 = 1, a_2 = 2, \ldots, a_{25} = 25$. Thus there were exactly 25 matches in the first 25 hours.

d) We need a different approach for this part, an approach, incidentally, that works for many numbers besides 30 in this setting. Let $a_1, a_2, \ldots, a_{75}$ be as before, and note that $1 \leq a_1 < a_2 < \cdots < a_{75} \leq 125$. By the pigeonhole principle two of the numbers among $a_1, a_2, \ldots, a_{31}$ are congruent modulo 30. If they differ by 30, then we have our solution. Otherwise they differ by 60 or more, so $a_{31} \geq 61$. Similarly, among $a_{31}$ through $a_{61}$, either we find a solution, or two numbers must differ by 60 or more; therefore we can assume that $a_{61} \geq 121$. But this means that $a_{66} \geq 126$, a contradiction.

44. Look at the pigeonholes $\{1000, 1001\}, \{1002, 1003\}, \{1004, 1005\}, \ldots, \{1098, 1099\}$. There are clearly 50 sets in this list. By the pigeonhole principle, if we have 51 numbers in the range from 1000 to 1099 inclusive, then at least two of them must come from the same set. These are the desired two consecutive house numbers.

46. Suppose this statement were not true. Then for each $i$, the $i^{th}$ box contains at most $n_i - 1$ objects. Adding, we have at most $(n_1 - 1) + (n_2 - 1) + \cdots + (n_t - 1) = n_1 + n_2 + \cdots + n_t - t$ objects in all, contradicting the fact that there were $n_1 + n_2 + \cdots + n_t - t + 1$ objects in all. Therefore the statement must be true.