26. A person can be born so as not to have the same birthday; then there are 1456 days on which the third person can be born so as not to have the same birthday as either of the first two, as so on, until there are 1468 – 4n days on which the n-th person can be born so as not to have the same birthday as any of the others. This gives a total of

\[ n \cdot 1460 \cdot 1456 \cdots (1468 - 4n) \]

ways in all. The other case is that in which there is no leap-year birthday. Then there are 1460 possible birthdays for the first person, 1456 for the second, and so on, down to 1464 – 4n for the n-th. Thus the total number of ways to choose birthdays without including February 29 is

\[ 1460 \cdot 1456 \cdots (1464 - 4n). \]

The sum of these two numbers is the numerator of the fraction giving the probability that all the birthdays are distinct. The denominator is 1461^n, since each person can have any birthday within the four-year cycle. Putting this all together, we see that the probability that there are at least two people with the same birthday is

\[ 1 - \frac{n \cdot 1460 \cdot 1456 \cdots (1468 - 4n) + 1460 \cdot 1456 \cdots (1464 - 4n)}{1461^n}. \]

24. There are 16 equally likely outcomes of flipping a fair coin five times in which the first flip comes up tails (each of the other flips can be either heads or tails). Of these only one will result in four heads appearing, namely THHHH. Therefore the answer is 1/16.

26. Intuitively the answer should be yes, because the parity of the number of 1’s is a fifty-fifty proposition totally determined by any one of the flips (for example, the last flip). What happened on the other flips is really rather irrelevant. Let us be more rigorous, though. There are 8 bit strings of length 3, and 4 of them contain an odd number of 1’s (namely 001, 010, 100, and 111). Therefore \( p(E) = 4/8 = 1/2 \). Since 4 bit strings of length 3 start with a 1 (namely 100, 101, 110, and 111), we see that \( p(F) = 4/8 = 1/2 \) as well. Furthermore, since there are 2 strings that start with a 1 and contain an odd number of 1’s (namely 100 and 111), we see that \( p(E \cap F) = 2/8 = 1/4 \). Then since \( p(E) \cdot p(F) = (1/2) \cdot (1/2) = 1/4 = p(E \cap F) \), we conclude from the definition that \( E \) and \( F \) are independent.

28. These questions are applications of the binomial distribution. Following the lead of King Henry VIII, we call having a boy success. Then \( p = 0.51 \) and \( n = 5 \) for this problem.

a) We are asked for the probability that \( k = 3 \). By Theorem 2 the answer is \( C(5, 3)0.51^30.49^2 \approx 0.32 \).

b) There will be at least one boy if there are not all girls. The probability of all girls is 0.49^5, so the answer is \( 1 - 0.49^5 \approx 0.972 \).

c) This is just like part (b): The probability of all boys is 0.51^5, so the answer is \( 1 - 0.51^5 \approx 0.965 \).

d) There are two ways this can happen. The answer is clearly \( 0.51^5 + 0.49^5 \approx 0.063 \).

30. a) The probability that all bits are a 1 is \((1/2)^{10} = 1/1024\). This is what is being asked for.

b) This is the same as part (a), except that the probability of a 1 bit is 0.6 rather than 1/2. Thus the answer is \( 0.6^{10} \approx 0.0060 \).

b) We need to multiply the probabilities of each bit being a 1, so the answer is

\[ \frac{1}{2} \cdot \frac{1}{2} \cdots \frac{1}{2^{10}} = \frac{1}{2^{1+2+\cdots+10}} = \frac{1}{2^{55}} \approx 2.8 \times 10^{-17}. \]

Note that this is essentially 0.