38. Assume that \( m \geq n \). Then each of the \( n \) vertices in one part has degree \( m \), and each of the \( m \) vertices in other part has degree \( n \). Thus the degree sequence is \( m, m, \ldots, m, n, n, \ldots, n \), where the sequence contains \( n \) copies of \( m \) and \( m \) copies of \( n \). We put the \( m \)'s first because we assumed that \( m \geq n \). If \( n \geq m \), then of course we would put the \( m \) copies of \( n \) first. If \( m = n \), this would mean a total of \( 2n \) copies of \( n \).

40. The 4-wheel (see Figure 5) with one edge along the rim deleted is such a graph. It has \((4+3+3+2+2)/2 = 7\) edges.

42. a) Since the number of odd-degree vertices has to be even, no graph exists with these degrees. Another reason no such graph exists is that the vertex of degree 0 would have to be isolated but the vertex of degree 5 would have to be adjacent to every other vertex, and these two statements are contradictory.

b) Since the number of odd-degree vertices has to be even, no graph exists with these degrees. Another reason no such graph exists is that the degree of a vertex in a simple graph is at most 1 less than the number of vertices.

c) A 6-cycle is such a graph. (See picture below.)

d) Since the number of odd-degree vertices has to be even, no graph exists with these degrees.

e) A 6-cycle with one of its diagonals added is such a graph. (See picture below.)

f) A graph consisting of three edges with no common vertices is such a graph. (See picture below.)

g) The 5-wheel is such a graph. (See picture below.)

h) Each of the vertices of degree 5 is adjacent to all the other vertices. Thus there can be no vertex of degree 1. So no such graph exists.

44. Since isolated vertices play no essential role, we can assume that \( d_u > 0 \). The sequence is graphic, so there is some simple graph \( G \) such that the degrees of the vertices are \( d_1, d_2, \ldots, d_n \). Without loss of generality, we can label the vertices of our graph so that \( d(v_1) = d_1 \). Among all such graphs, choose \( G \) to be one in which \( v_1 \) is adjacent to as many of \( v_2, v_3, \ldots, v_{d_1+1} \) as possible. (The worst case might be that \( v_1 \) is not adjacent to any of these vertices.) If \( v_1 \) is adjacent to all of them, then we are done. We will show that if there is a vertex among \( v_2, v_3, \ldots, v_{d_1+1} \) that \( v_1 \) is not adjacent to, then we can find another graph with \( d(v_1) = d_i \) and having \( v_1 \) adjacent to one more of the vertices \( v_2, v_3, \ldots, v_{d_1+1} \) than is true for \( G \). This is a contradiction to the choice of \( G \), and hence we will have shown that \( G \) satisfies the desired condition.

Under this assumption, then, let \( u \) be a vertex among \( v_2, v_3, \ldots, v_{d_1+1} \) that \( v_1 \) is not adjacent to, and let \( w \) be a vertex not among \( v_2, v_3, \ldots, v_{d_1+1} \) that \( v_1 \) is adjacent to; such a vertex \( w \) has to exist because \( d(v_1) = d_1 \). Because the degree sequence is listed in nonincreasing order, we have \( d(u) \geq d(w) \). Consider all the vertices that are adjacent to \( u \). It cannot be the case that \( w \) is adjacent to each of them, because then \( w \) would have a higher degree than \( u \) (because \( w \) is adjacent to \( v_1 \) as well, but \( u \) is not). Therefore there is some vertex \( x \) such that edge \( ux \) is present but edge \( xw \) is not present. Note also that edge \( v_1w \) is present but edge \( v_1u \) is not present. Now construct the graph \( G' \) to be the same as \( G \) except that edges \( ux \) and \( v_1w \) are removed and edges \( xw \) and \( v_1u \) are added. The degrees of all vertices are unchanged, but this graph has \( v_1 \) adjacent to more of the vertices among \( v_2, v_3, \ldots, v_{d_1+1} \) than is the case in \( G \). That gives the desired contradiction, and our proof is complete.

46. Given a sequence \( d_1, d_2, \ldots, d_n \), if \( n = 2 \), then the sequence is graphic if and only if \( d_1 = d_2 = 1 \) (the graph consists of one edge)—this is one base case. Otherwise, if \( n < d_1 + 1 \), then the sequence is not graphic—this