22. An asymmetric relation must be antisymmetric, since the hypothesis of the condition for antisymmetry is false if the relation is asymmetric. The relation $\{(a,a)\}$ on $\{a\}$ is antisymmetric but not asymmetric, however, so the answer to the second question is no. In fact, it is easy to see that $R$ is asymmetric if and only if $R$ is antisymmetric and irreflexive.

24. Of course many answers are possible. The empty relation is always asymmetric ($x$ is never related to $y$). A less trivial example would be $(a,b) \in R$ if and only if $a$ is taller than $b$. Clearly it is impossible that both $a$ is taller than $b$ and $b$ is taller than $a$ at the same time.

26. a) $R^{-1} = \{(b,a) \mid (a,b) \in R\} = \{(b,a) \mid a < b\} = \{(a,b) \mid a > b\}$
   b) $\overline{R} = \{(a,b) \mid (a,b) \notin R\} = \{(a,b) \mid a \neq b\} = \{(a,b) \mid a \geq b\}$

28. a) Since this relation is symmetric, $R^{-1} = R$.
   b) This relation consists of all pairs $(a,b)$ in which state $a$ does not border state $b$.

30. These are merely routine exercises in set theory. Note that $R_1 \subseteq R_2$.
   a) $\{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (3,4)\} = R_2$
   b) $\{(1,2), (2,3), (3,4)\} = R_3$
   c) $\emptyset$
   d) $\{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\}$

32. Since $(1,2) \in R$ and $(2,1) \in S$, we have $(1,1) \in S \circ R$. We use similar reasoning to form the rest of the pairs in the composition, giving us the answer $\{(1,1), (1,2), (2,1), (2,2)\}$.

34. a) The union of two relations is the union of these sets. Thus $R_1 \cup R_3$ holds between two real numbers if $R_1$ holds or $R_3$ holds (or both, it goes without saying). Here this means that the first number is greater than the second or vice versa—in other words, that the two numbers are not equal. This is just relation $R_6$.
   b) For $(a,b)$ to be in $R_3 \cup R_6$, we must have $a > b$ or $a = b$. Since this happens precisely when $a \geq b$, we see that the answer is $R_2$.
   c) The intersection of two relations is the intersection of these sets. Thus $R_2 \cap R_4$ holds between two real numbers if $R_2$ holds and $R_4$ holds as well. Thus for $(a,b)$ to be in $R_2 \cap R_4$, we must have $a \geq b$ and $a \leq b$.
   d) Since this happens precisely when $a = b$, we see that the answer is $R_5$.
   e) For $(a,b)$ to be in $R_3 \cap R_5$, we must have $a < b$ and $a = b$. It is impossible for $a < b$ and $a = b$ to hold at the same time, so the answer is $\emptyset$, i.e., the relation that never holds.
   f) Recall that $R_1 - R_2 = R_1 \cap \overline{R_2}$. But $\overline{R_2} = R_3$, so we are asked for $R_1 \cap R_3$. It is impossible for $a > b$ and $a < b$ to hold at the same time, so the answer is $\emptyset$, i.e., the relation that never holds.
   g) Reasoning as in part (f), we want $\overline{R_2} \cap \overline{R_1} = R_2 \cap R_4$, which is $R_5$ (this was part (c)).
   h) Recall that $R_2 \oplus R_4 = (R_2 \cap \overline{R_4}) \cup (R_4 \cap \overline{R_2})$. We see that $R_1 \cap \overline{R_5} = R_1 \cap R_2 = R_1$, and $R_3 \cap \overline{R_1} = R_3 \cap R_4 = R_3$. Thus our answer is $R_1 \cup R_3 = R_6$ (as in part (a)).

36. Recall that the composition of two relations all defined on a common set is defined as follows: $(a,c) \in S \circ R$ if and only if there is some element $b$ such that $(a,b) \in R$ and $(b,c) \in S$. We have to apply this in each case.
   a) For $(a,c)$ to be in $R_1 \circ R_1$, we must find an element $b$ such that $(a,b) \in R_1$ and $(b,c) \in R_1$. This means that $a > b$ and $b > c$. Clearly this can be done if and only if $a > c$ to begin with. But that is precisely the statement that $(a,c) \in R_1$. Therefore we have $R_1 \circ R_1 = R_1$. We can interpret (part of) this as showing that $R_1$ is transitive.