1. Write the negation of the statement in good English. Don’t write “It is not true that . . . .”

(a) Some bananas are yellow

Solution: No bananas are yellow

(b) All integers ending in the digit 7 are odd.

Solution: There exists an integer ending in the digit 7 that is even

(c) No tests are easy.

Solution: There are easy tests

(d) Roses are red and violets are blue

Solution: Roses are not red or violets are not blue

(e) Some skiers do not speak Spanish

Solution: All skiers speak Spanish
2. Suppose \( p, q \) and \( r \) are proposition variables. That is, they are true or false. Using proposition logic, write a proposition that is true when \( p \) and \( q \) are true and \( r \) is false, but false otherwise. You may use any or all of the logical operators \( \lor, \land, \neg, \rightarrow \). Show that your proposition gives the desired result by writing a truth table.

**Solution:**

\[(p \land q) \land \neg r\]

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3. Suppose the variable $x$ represents students and $y$ represents courses, and

- $F(x) : x$ is a freshman
- $A(x) : x$ is a part-time student
- $T(x, y) : x$ is taking $y$

Write the following statements in good English without using variables in your answers.

(a) $F($Mikko$)$

**Solution:** Mikko is a freshman

(b) $\neg \exists y T($Joe, $y$)

**Solution:** There is no class that Joe is taking.

(c) $\exists x (A(x) \land \neg F(x))$

**Solution:** There is a part-time student who is not a freshman.

4. Suppose the variable $x$ represents students and the variable $y$ represents courses, and

- $T(x, y) : x$ is taking $y$
- $P(x, y) : x$ passed $y$

Write the statement in good English. Do not use variables in your answers.

(a) $\neg P($John, CSE 101$)$

**Solution:** John did not pass CSE 101

(b) $\exists y \forall x T(x, y)$

**Solution:** There is a course all students are taking.
(c) $\forall x \exists y T(x, y)$

**Solution:** Every student is taking some course

5. Is the following a valid argument? If $n$ is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$. Explain your answer.

   **Solution:** Not valid, fallacy of affirming the conclusion.

6. Given any 40 people, prove that at least four of them were born in the same month of the year.

   **Solution:** Assume not for sake of contradiction. Then there are most three people born in each month. Then there are at most $3 \cdot (12) = 36$ people, a contradiction.
7. Prove that the following is true for all positive integers \( n \): \( n \) is even if and only if \( 3n^2 + 8 \) is even.

**Solution:** We need to prove both directions since it is an if and only if.

Say \( n \) is even. Then \( n = 2k \) for some fixed integer \( k \). Hence,

\[
3((2k)^2) + 8 = 12k^2 + 8 = 2(6k^2 + 4)
\]

Thus, \( n^2 \) must be even when \( n \) is even. It remains to show that if \( 3n^2 + 8 \) is even then \( n \) is even. To do this, we use the contrapositive. We will show that if \( n \) is odd then \( 3n^2 + 8 \) is odd. By assumption, \( n \) is odd so we can write \( n = 2k + 1 \) for some fixed integer \( k \). Now we have that,

\[
3((2k + 1)^2) + 8 = 12k^2 + 12k + 11 = 2(6k^2 + 6k + 5) + 1
\]

Thus, \( n^2 \) is odd, proving the contrapositive.
8. Prove that \((q \land (p \rightarrow \lnot q)) \rightarrow \lnot p\) is a tautology using propositional equivalence. Do not use a truth table. (Hint: Replace the first ‘implies’ before the second)

Solution:

\[
(q \land (p \rightarrow \lnot q)) \rightarrow \lnot p \\
\Rightarrow (q \land (\lnot p \lor \lnot q)) \rightarrow \lnot p \\
\Rightarrow ((q \land \lnot p) \lor (q \land \lnot q)) \rightarrow \lnot p \\
\Rightarrow (q \land \lnot p) \rightarrow \lnot p \\
\Rightarrow \lnot (q \land \lnot p) \land \lnot p \\
\Rightarrow (\lnot q \lor \lnot \lnot p) \land \lnot p \\
\Rightarrow (\lnot q \lor p) \land \lnot p \\
\Rightarrow \lnot q \lor (p \land \lnot p) \\
\Rightarrow \lnot q \lor T \\
\Rightarrow T
\]
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