Exam 1
1. Write the negation of the statement in good English. Don’t write “It is not true that . . . .”

(a) Some bananas are yellow

(b) All integers ending in the digit 7 are odd.

(c) No tests are easy.

(d) Roses are red and violets are blue

(e) Some skiers do not speak Spanish
2. Suppose $p$, $q$ and $r$ are proposition variables. That is, they are true or false. Using proposition logic, write a proposition that is true when $p$ and $q$ are true and $r$ is false, but false otherwise. You may use any or all of the logical operators $\lor$, $\land$, $\neg$, $\rightarrow$. Show that your proposition gives the desired result by writing a truth table.
3. Suppose the variable \( x \) represents students and \( y \) represents courses, and

- \( F(x) : x \) is a freshman
- \( A(x) : x \) is a part-time student
- \( T(x, y) : x \) is taking \( y \)

Write the following statements in good English without using variables in your answers.

(a) \( F(\text{Mikko}) \)

(b) \( \neg \exists y T(\text{Joe}, y) \)

(c) \( \exists x (A(x) \land \neg F(x)) \)

4. Suppose the variable \( x \) represents students and the variable \( y \) represents courses, and

- \( T(x, y) : x \) is taking \( y \)
- \( P(x, y) : x \) passed \( y \)

Write the statement in good English. Do not use variables in your answers.

(a) \( \neg P(\text{John, CSE 101}) \)

(b) \( \exists y \forall x T(x, y) \)

(c) \( \forall x \exists y T(x, y) \)
5. Is the following a valid argument? If $n$ is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$. Explain your answer.

6. Given any 40 people, prove that at least four of them were born in the same month of the year.
7. Prove that the following is true for all positive integers $n$: $n$ is even if and only if $3n^2 + 8$ is even.
8. Prove that \((q \land (p \to \neg q)) \to \neg p\) is a tautology using propositional equivalence. Do not use a truth table. (Hint: Replace the first ‘implies’ before the second)
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