Exam 3 Extra Problems
1. Say that a 'word' contains seven letters from the alphabet. How many words begin with A or B?

2. How many words begin with A or B and end with A or B?

3. How many words have no vowels?

4. How many bit strings of length 10 have exactly six 0's?

5. Find the number of permutations of the letters in the word TATTERED.

6. What is the probability that a fair coin lands Heads 6 times in a row?

7. Prove or disprove: $p(E \cup F) = p(E) + p(F)$ for all events $E$ and $F$.

8. A group of ten women and ten men are in a room. If five of the 20 are selected at random and put in a row for a picture, what is the probability that the five are of the same sex?

9. Suppose $A$ is the set composed of all ordered pairs of positive integers. That is, $(a, b)$ is an element of $A$ if $a$ and $b$ are integers. Let $R$ be the relation defined on $A$ where $(a, b)R(c, d)$ means that $a + d = b + c$. Prove that $R$ is an equivalence relation.

10. How can you determine if a graph has a non-simple cycle that visits every edge exactly one and starts and ends at the same vertex?

11. A simple graph is regular if every vertex has the same degree. For which positive integers $n$ are the following graphs regular: $C_n$, $W_n$, $K_n$, $Q_n$?

12. Give a recurrence relation for $e_n$, the number of edges of the graph $W_n$.

13. Find all string recognized by this deterministic finite-state automaton.

14. Construct a finite-state automaton that recognizes the set represented by the regular expression $10^*$.

15. Consider any walk from $u$ to $v$ in a simple graph, with $u \neq v$ and no edge crossed more than once. A walk is a sequence of vertices which are connected. Let $E'$ be the set of edges on this walk. Consider a new graph on the original set of vertices and only the edges $E'$ from the walk. Prove that the only vertices with odd degree are $u$ and $v$. 