1. Devise a recursive algorithm to find $a^{2^n}$, where $a$ is a real number and $n$ is a positive integer. [Hint: Use the equality $a^{2^{n+1}} = (a^{2^n})^2$.]

2. Give a recursive algorithm for finding the sum of the first $n$ positive integers.

3. Let $x$ and $y$ be real numbers. Consider the program

   
   \[
   \text{if } x < y \\
   \quad \text{min} = x \\
   \text{else} \\
   \quad \text{min} = y
   \]

   Prove that $((x \leq y) \land \text{min} = x) \lor ((x > y) \land \text{min} = y)$ is true after this code is executed. Be careful to consider the case where $x = y$.

4. Give an example of a function from the set of integers to the set of integers that is
   
   (a) one-to-one but not onto.
   (b) onto but not one-to-one.
   (c) both onto and one-to-one (but different from the identity function).
   (d) neither one-to-one nor onto.

5. Give a recursive algorithm for finding a mode of a list of integers. (A mode is an element in the list that occurs at least as often as every other element.)