1. Devise a recursive algorithm to find \( a^{2^n} \), where \( a \) is a real number and \( n \) is a positive integer. [Hint: Use the equality \( a^{2^{n+1}} = (a^{2^n})^2 \).]

2. Give a recursive algorithm for finding the sum of the first \( n \) positive integers.

3. Let \( x \) and \( y \) be real numbers. Consider the program

\[
\begin{align*}
\text{if } x < y & \quad \text{min} = x \\
\text{else} & \quad \text{min} = y
\end{align*}
\]

Prove that \(((x \leq y) \land \text{min} = x) \lor ((x > y) \land \text{min} = y)\) is true after this code is executed. Be careful to consider the case where \( x = y \).

5. Give an example of a function from the set of integers to the set of integers that is

(a) one-to-one but not onto.
(b) onto but not one-to-one.
(c) both onto and one-to-one (but different from the identity function).
(d) neither one-to-one nor onto.

6. Give a recursive algorithm for finding a mode of a list of integers. (A mode is an element in the list that occurs at least as often as every other element.)