1. Suppose that $X$ and $Y$ are random variables and that $X$ and $Y$ are nonnegative for all points in a sample space $S$. Let $Z$ be the random variable defined by $Z(s) = \max(X(s), Y(s))$ for all elements $s \in S$. Show that $E(Z) \leq E(X) + E(Y)$.

2. What is the variance of the number of times a 6 appears when a fair die is rolled 10 times?

3. Provide an example that shows that the variance of the sum of two random variables is not necessarily equal to the sum of their variances when the random variables are not independent.

4. Suppose that we roll a fair die until a 6 comes up.
   (a) What is the probability that we roll the die $n$ times?
   (b) What is the expected number of times we roll the die?

5. Consider randomly generating a bit string of length three. Each bit is independently 1 or 0 with equal probability. Let $E$ be the event that the string contains an odd number of 1s and let $F$ be the event that the string starts with a 1. Are $E$ and $F$ independent? Give a full proof.