Announcements

- Homework today!
- Exam next Wednesday!
  - Review session Monday
Permutations

A permutation is an ordered arrangement of elements from a set.

On ordered arrangement of \( r \) elements of a set is called an \( r \)-permutation.

The number of \( r \)-permutations of a set with \( n \) elements is \( P(n,r) \).

If \( n \) and \( r \) are integers with \( 0 \leq r \leq n \), then \( P(n,r) = \frac{n!}{(n-r)!} \)
Permutations

In a running race of 12 sprinters, each of the top 5 finishers receives a different medal.

How many ways are there to award the 5 medals?

\[
P(n,r) = \frac{n!}{(n-r)!}
\]

a) 60
b) \(12^5\)
c) \(12!/7!\)
d) \(5^{12}\)
e) No clue

http://www.mathsisfun.com/combinatorics/combinations-permutations-calculator.html
Permutations

Suppose you have time to listen to 10 songs on your daily jog around campus. There are 6 Cold War Kids tunes, 8 Iggy Pop tunes, and 3 Velvet Underground tunes to choose from.

How many different jog playlists can you make?

\[ P(17,10) \]
Suppose you have time to listen to 10 songs on your daily jog around campus. There are 6 Cold War Kids tunes, 8 Iggy Pop tunes, and 3 Velvet Underground tunes to choose from.

Now suppose you want to listen to 4 Cold War Kids, 4 Iggy Pop, and 2 Velvet Underground tunes, in that band order. How many playlists can you make?

\[ P(6,4) \times P(8,4) \times P(3,2) \]
Permutations

Suppose you have time to listen to 10 songs on your daily jog around campus. There are 6 Cold War Kids tunes, 8 Iggy Pop tunes, and 3 Velvet Underground tunes to choose from.

Finally, suppose you still want 4 Cold War Kids, 4 Iggy Pop, and 2 Velvet Underground tunes, and the order of the groups does not matter, but you get dizzy and fall down if all the songs by any one group are not played together.

How many playlists are there now?

\[ P(6,4) \times P(8,4) \times P(3,2) \times 3! \]
Permutations

In how many ways can 5 distinct Martians and 3 distinct Jovians stand in line, if no two Jovians stand together?

\[ 5! \times P(6,3) \]
Combinations

A combination is an unordered selection of elements from some set.

The number of combinations of $r$ distinct objects chosen from $n$ distinct objects is denoted by $C(n,r)$ or $nCr$ or $\binom{n}{r}$, and is read “$n$ choose $r$.”

\[
C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{((n-r)!r!)}
\]
Combination

A basketball squad consists of 12 players, 5 of which make up a team. How many different teams of players can you make from the 12?

What’s the difference?

In a running race of 12 sprinters, each of the top 5 finishers receives a different medal. How many ways are there to award the 5 medals?

\[ \text{C}(12,5) = \text{P}(12,5) = \text{C}(12,5) \times 5! \]
Combinations

A committee of 3 students is to be selected from a class consisting of 4 freshmen, and 5 sophomores.

In how many ways can a committee with at most 1 freshman be selected?

Start with a easier question first...

What is the total number of possible combinations to select a committee of 3 students?

Universe  = C(9,3) = 84
# Combinations

Universe  = \( C(9,3) = 84 \)

<table>
<thead>
<tr>
<th>#Fresh</th>
<th>Comb.</th>
<th>#Soph</th>
<th>Comb.</th>
<th>Both Parts</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( C(4,0) )</td>
<td>3</td>
<td>( C(5,3) )</td>
<td>( C(4,0) \times C(5,3) )</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>( C(4,1) )</td>
<td>2</td>
<td>( C(5,2) )</td>
<td>( C(4,1) \times C(5,2) )</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>( C(4,2) )</td>
<td>1</td>
<td>( C(5,1) )</td>
<td>( C(4,2) \times C(5,1) )</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>( C(4,3) )</td>
<td>0</td>
<td>( C(5,0) )</td>
<td>( C(4,3) \times C(5,0) )</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>Total Combinations</strong></td>
<td>84</td>
</tr>
</tbody>
</table>

In how many ways can a committee with at most 1 freshman be selected?  \( 10 + 40 = 50 \)

In how many ways can a committee with at least 1 freshman be selected?  \( 40 + 30 + 4 = 74 \)
Combinations

A committee of 8 students is to be selected from a class consisting of 19 freshmen, and 34 sophomores.

In how many ways can 3 freshmen and 5 sophomores be selected?

\[ C(19,3) \times C(34,5) \]
Combinations

A committee of 8 students is to be selected from a class consisting of 19 freshmen, and 34 sophomores.

In how many ways can a committee with exactly 1 freshman be selected?

\[ C(19,1) \times C(34,7) \]
Combinations

A committee of 8 students is to be selected from a class consisting of 19 freshmen, and 34 sophomores.

In how many ways can a committee with at most 1 freshman be selected?

\[ C(34,7) \times C(19+27,1) = 537616 \times 46 = 24,746,236 \]

Break it into cases:

\[ C(34,8) \times C(19,0) + C(34,7) \times C(19,1) = 18,156,204 \times 1 + 5379616 \times 19 = 120,368,908 \]
Combinations with repetition

Suppose you want to buy 5 bags of chips from the 3 kinds you like. In how many different ways can you stock up?

Out of 7 items, we are choosing 2 to be bars.
From that, and our understanding of the model, we can report the answer.

\[ C(7,2) = C(7,5) \]
Combinations with repetition

There are \( C(n+r-1,r) \) \( r \)-sized combinations from a set of \( n \) elements when repetition is allowed.

- Example: How many solutions are there to the equation

\[
\begin{align*}
\sum_{i=1}^{4} x_i &= 10
\end{align*}
\]

- When the variables are nonnegative integers?

\[
1 + 3 + 6 + 0 = 10
\]
Permutations with indistinguishable objects

How many different strings can be made from the letters in the word rat?

\[ C(3,1) \times C(2,1) \times C(1,1) = 6 \]

How many different strings can be made from the letters in the word egg?

\[ C(3,2) \times C(1,1) = 3 \]
Permutations with indistinguishable objects

• How many different strings can be made from the letters in the phrase nano-nano?

• Key thoughts: 8 positions, 3 kinds of letters to place.
• In how many ways can we place the ns?
• In how many ways can we place the as?
• In how many ways can we place the os?

\[
\begin{align*}
\binom{8}{4} \binom{4}{2} \binom{2}{2} &= \frac{8!}{4!4!} \cdot \frac{4!}{2!2!} \cdot \frac{2!}{2!0!} = \frac{8!}{4!2!2!} \\
\end{align*}
\]
Extra Examples
Permutations with indistinguishable objects

- How many distinct permutations are there of the letters in the word APALACHICOLA?

- How many if the two Ls must appear together?

- How many if the first letter must be an A?
Permutations with indistinguishable objects

• How many distinct permutations are there of the letters in the word APALACHICOLA?

\[
\frac{12!}{4!2!2!}
\]

• How many if the two Ls must appear together?

\[
\frac{11!}{4!2!}
\]

• How many if the first letter must be an A?

\[
\frac{11!}{3!2!2!}
\]
Group Problems

1) Five people get into an elevator. There are seven floors on which they can get off. What is the total number of different ways they can do this?

2) Find the number of integers between 1 and 400 (inclusive) that are divisible by 6 and 8

3) Find the number of integers (between 1 and 400) that are divisible by 6 or 8