One of the problems below will be chosen at random in class for a quiz.

1. Determine whether each of these sets is the power set of a set, where a and b are distinct elements.
   
   (a) $\emptyset$
   
   (b) $\{\emptyset, \{a\}\}$
   
   (c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$
   
   (d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

2. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

   (a) the set of people who speak English, the set of people who speak English with an Australian accent
   
   (b) the set of fruits, the set of citrus fruits
   
   (c) the set of students studying discrete mathematics, the set of students studying data structures

3. Suppose that you know that a golfer plays the first hole of a golf course with an infinite number of holes and that if this golfer plays one hole, then the golfer goes on to play the next hole. Prove that this golfer plays every hole on the course.

4. Let $P(n)$ be the statement that $\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \ldots + n^3 = (n(n+1)/2)^2$ for the positive integer $n$.

   (a) What is the statement $P(1)$?
   
   (b) Show that $P(1)$ is true, completing the basis step of the proof.
   
   (c) What is the inductive hypothesis?
   
   (d) What do you need to prove in the inductive step?
   
   (e) Complete the inductive step, identifying where you use the inductive hypothesis.
   
   (f) Explain why these steps show that this formula is true whenever $n$ is a positive integer.

5. Suppose that $A$ is the set of sophomores at your school and $B$ is the set of students in discrete mathematics at your school. Express each of these sets in terms of $A$ and $B$.

   (a) the set of sophomores taking discrete mathematics in your school
   
   (b) the set of sophomores at your school who are not taking discrete mathematics
   
   (c) the set of students at your school who either are sophomores or are taking discrete mathematics
(d) the set of students at your school who either are not sophomores or are not taking discrete mathematics

6. Show that if $A$ and $B$ are sets with $A \subseteq B$

   (a) $A \cup B = B$

   (b) $A \cap B = A$