40. This is an identity; each side consists of those things that are in an odd number of the sets \( A, B, \) and \( C \).

42. This is an identity; each side consists of those things that are in an odd number of the sets \( A, B, C, \) and \( D \).

44. A finite set is a set with \( k \) elements for some natural number \( k \). Suppose that \( A \) has \( n \) elements and \( B \) has \( m \) elements. Then the number of elements in \( A \cup B \) is at most \( n + m \) (it might be less because \( A \cap B \) might be nonempty). Therefore by definition, \( A \cup B \) is finite.

46. To count the elements of \( A \cup B \cup C \) we proceed as follows. First we count the elements in each of the sets and add. This certainly gives us all the elements in the union, but we have overcounted. Each element in \( A \cap B, A \cap C, \) and \( B \cap C \) has been counted twice. Therefore we subtract the cardinalities of these intersections to make up for the overcount. Finally, we have compensated a bit too much, since the elements of \( A \cap B \cap C \) have now been counted three times and subtracted three times. We adjust by adding back the cardinality of \( A \cap B \cap C \).

48. We note that these sets are increasing, that is, \( A_1 \subseteq A_2 \subseteq A_3 \subseteq \cdots \). Therefore, the union of any collection of these sets is just the one with the largest subscript, and the intersection is just the one with the smallest subscript.

a) \( A_n = \{\ldots, -2, -1, 0, 1, \ldots, n\} \)  

b) \( A_1 = \{\ldots, -2, -1, 0, 1\} \)

50. a) As \( i \) increases, the sets get smaller: \( \cdots \subseteq A_3 \subseteq A_2 \subseteq A_1 \). All the sets are subsets of \( A_1 \), which is the set of positive integers, \( \mathbb{Z}^+ \). It follows that \( \bigcup_{i=1}^{\infty} A_i = \mathbb{Z}^+ \). Every positive integer is excluded from at least one of the sets (in fact from infinitely many), so \( \bigcap_{i=1}^{\infty} A_i = \emptyset \).

b) All the sets are subsets of the set of natural numbers \( \mathbb{N} \) (the nonnegative integers). The number 0 is in each of the sets, and every positive integer is in exactly one of the sets, so \( \bigcup_{i=1}^{\infty} A_i = \mathbb{N} \) and \( \bigcap_{i=1}^{\infty} A_i = \{0\} \).

c) As \( i \) increases, the sets get larger: \( A_1 \subseteq A_2 \subseteq A_3 \cdots \). All the sets are subsets of the set of positive real numbers \( \mathbb{R}^+ \), and every positive real number is included eventually, so \( \bigcup_{i=1}^{\infty} A_i = \mathbb{R}^+ \). Because \( A_1 \) is a subset of each of the others, \( \bigcap_{i=1}^{\infty} A_i = A_1 = (0, 1) \) (the interval of all real numbers between 0 and 1, exclusive).

D) This time, as in part (a), the sets are getting smaller as \( i \) increases: \( \cdots \subseteq A_3 \subseteq A_2 \subseteq A_1 \). Because \( A_1 \) includes all the others, \( \bigcup_{i=1}^{\infty} A_i = (1, \infty) \) (all real numbers greater than 1). Every number eventually gets excluded as \( i \) increases, so \( \bigcap_{i=1}^{\infty} A_i = \emptyset \). Notice that \( \infty \) is not a real number, so we cannot write \( \bigcap_{i=1}^{\infty} A_i = \{\infty\} \).

52. a) 00 1110 0000  

b) 10 1001 0001  

c) 01 1100 1110

54. a) No elements are included, so this is the empty set.

b) All elements are included, so this is the universal set.

56. The bit string for the symmetric difference is obtained by taking the bitwise exclusive \( OR \) of the two bit strings for the two sets, since we want to include those elements that are in one set or the other but not both.

58. We can take the bitwise \( OR \) (for union) or \( AND \) (for intersection) of all the bit strings for these sets.

60. The successor set has one more element than the original set, namely the original set itself. Therefore the answer is \( n + 1 \).