12. We argue directly by showing that if the hypothesis is true, then so is the conclusion. An alternative approach, which we show only for part (a), is to use the equivalences listed in the section and work symbolically.

a) Assume the hypothesis is true. Then p is false. Since $p \lor q$ is true, we conclude that q must be true. Here is a more “algebraic” solution: $\neg p \land (p \lor q) \rightarrow q \equiv \neg p \lor (p \lor q) \lor q \equiv p \lor \neg (p \lor q) \lor q \equiv (p \lor q) \lor q \equiv T$. The reasons for these logical equivalences are, respectively, Table 7, line 1; De Morgan’s law; double negation; commutative and associative laws; negation law.

b) We want to show that if the entire hypothesis is true, then the conclusion $p \rightarrow r$ is true. To do this, we need only show that if $p$ is true, then $r$ is true. Suppose $p$ is true. Then by the first part of the hypothesis, we conclude that $q$ is true. It now follows from the second part of the hypothesis that $r$ is true, as desired.

c) Assume the hypothesis is true. Then $p$ is true, and since the second part of the hypothesis is true, we conclude that $q$ is also true, as desired.

d) Assume the hypothesis is true. Since the first part of the hypothesis is true, we know that either $p$ or $q$ is true. If $p$ is true, then the second part of the hypothesis tells us that $r$ is true; similarly, if $q$ is true, then the third part of the hypothesis tells us that $r$ is true. Thus in each case we conclude that $r$ is true.

14. This is not a tautology. It is saying that knowing that the hypothesis of an conditional statement is false allows us to conclude that the conclusion is also false, and we know that this is not valid reasoning. To show that it is not a tautology, we need to find truth assignments for $p$ and $q$ that make the entire proposition false. Since this is possible only if the conclusion if false, we want to let $q$ be true; and since we want the hypothesis to be true, we must also let $p$ be false. It is easy to check that if, indeed, $p$ is false and $q$ is true, then the conditional statement is false. Therefore it is not a tautology.

16. The first of these propositions is true if and only if $p$ and $q$ have the same truth value. The second is true if and only if either $p$ and $q$ are both true, or $p$ and $q$ are both false. Clearly these two conditions are saying the same thing.

18. It is easy to see from the definitions of conditional statement and negation that each of these propositions is false in the case in which $p$ is true and $q$ is false, and true in the other three cases. Therefore the two propositions are logically equivalent.

20. It is easy to see from the definitions of the logical operations involved here that each of these propositions is true in the cases in which $p$ and $q$ have the same truth value, and false in the cases in which $p$ and $q$ have opposite truth values. Therefore the two propositions are logically equivalent.

22. Suppose that $(p \land q) \rightarrow (p \land r)$ is true. We want to show that $p \rightarrow (q \land r)$ is true, which means that we want to show that $q \land r$ is true whenever $p$ is true. If $p$ is true, since we know that both $p \rightarrow q$ and $p \rightarrow r$ are true from our assumption, we can conclude that $q$ is true and that $r$ is true. Therefore $q \land r$ is true, as desired. Conversely, suppose that $p \rightarrow (q \land r)$ is true. We need to show that $p \rightarrow q$ is true and that $p \rightarrow r$ is true, which means that if $p$ is true, then so are $q$ and $r$. But this follows from $p \rightarrow (q \land r)$.