Announcements

• Homework 1 is posted online and due next Thursday
Homework 1

• **Problems due next Thursday Jan. 22\textsuperscript{rd}**
  – HW and Quiz problems are also posted online

• **Quiz Tuesday Jan. 27th**
  – I will choose one problem from this set
Apples, Oranges, and bags...Oh my!
Propositional Logic (Section 1)

- Proofs involve stepping through a mathematical argument
- Propositional Logic provides such steps
- Today we will discuss the process of moving from one proposition to the next to form a mathematical argument
Logical Equivalence

- Challenge: Try to find a proposition that is equivalent to $p \rightarrow q$, but that uses only the connectives $\neg$, $\land$, and $\lor$

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- $p \rightarrow q$ is logically equivalent to $\neg p \lor q$
- or $p \rightarrow q \equiv \neg p \lor q$
Are these equivalent?

- **Contrapositives**: \( p \rightarrow q \) and \( \neg q \rightarrow \neg p \)
  
  Ex. “If it is noon, then I am hungry.”
  “If I am not hungry, then it is not noon.”

- **Converses**: \( p \rightarrow q \) and \( q \rightarrow p \)
  
  Ex. “If it is noon, then I am hungry.”
  “If I am hungry, then it is noon.”

- **Inverses**: \( p \rightarrow q \) and \( \neg p \rightarrow \neg q \)
  
  Ex. “If it is noon, then I am hungry.”
  “If it is not noon, then I am not hungry.”

Let’s take a vote
Are these equivalent?

- **Contrapositives:** \( p \rightarrow q \equiv \neg q \rightarrow \neg p \) ?
  Ex. “If it is noon, then I am hungry.”
  “If I am not hungry, then it is not noon.”

- **Converses:** \( p \rightarrow q \equiv q \rightarrow p \) ?
  Ex. “If it is noon, then I am hungry.”
  “If I am hungry, then it is noon.

- **Inverses:** \( p \rightarrow q \equiv \neg p \rightarrow \neg q \) ?
  Ex. “If it is noon, then I am hungry.”
  “If it is not noon, then I am not hungry.”

A) If just 1 is True
B) If just 2 is True
C) If just 3 is True
D) If both 1 & 2 are True
E) If both 2 & 3 are True
F) If both 1 & 3 are True
G) If All are True
H) If none are True
I) I don’t understand why there are so many choices... 😐
Are these equivalent?

- **Contrapositives:** \( p \rightarrow q \equiv \neg q \rightarrow \neg p \)?
  - Yes.
  - Ex. “If it is noon, then I am hungry.”
    - “If I am not hungry, then it is not noon.”

- **Converses:** \( p \rightarrow q \equiv q \rightarrow p \)?
  - No.
  - Ex. “If it is noon, then I am hungry.”
    - “If I am hungry, then it is noon.”

- **Inverses:** \( p \rightarrow q \equiv \neg p \rightarrow \neg q \)?
  - No.
  - Ex. “If it is noon, then I am hungry.”
    - “If it is not noon, then I am not hungry.”
A **tautology** is a proposition that’s always TRUE.

A **contradiction** is a proposition that’s always FALSE.

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Logical Equivalences

- **Identity**
  - $p \land T \equiv p$
  - $p \lor F \equiv p$

- **Domination**
  - $p \lor T \equiv T$
  - $p \land F \equiv F$

- **Idempotent**
  - $p \lor p \equiv p$
  - $p \land p \equiv p$

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Logical Equivalences Continued

• **Excluded Middle** \[ p \lor \neg p \equiv T \]

• **Uniqueness** \[ p \land \neg p \equiv F \]

• **Double negation** \[ \neg(\neg p) \equiv p \]
Logical Equivalences Continued

- **Commutativity**
  \[ p \lor q \equiv q \lor p \]
  \[ p \land q \equiv q \land p \]

- **Associativity**
  \[ (p \lor q) \lor r \equiv p \lor (q \lor r) \]
  \[ (p \land q) \land r \equiv p \land (q \land r) \]

- **Distributivity**
  \[ p \lor (q \land r) \equiv (p \lor q) \land (p \lor r) \]
  \[ p \land (q \lor r) \equiv (p \land q) \lor (p \land r) \]
Proof of Distributivity

\[ p \lor (q \land r) \equiv (p \lor q) \land (p \lor r) \]

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DeMorgan’s

- **De Morgan’s I** \( \neg (p \lor q) \equiv \neg p \land \neg q \)

- **De Morgan’s II** \( \neg (p \land q) \equiv \neg p \lor \neg q \)
Example of De Morgan’s
De Morgan’s Continued

- **De Morgan’s II**

\[ \neg(p \land q) \equiv \neg p \lor \neg q \]

\[ \neg(p \land q) \equiv \neg(\neg\neg p \land \neg\neg q) \quad \text{Double negation} \]

\[ \equiv \neg\neg(\neg p \lor \neg q) \quad \text{DeMorgan’s I} \]

\[ \equiv (\neg p \lor \neg q) \quad \text{Double negation} \]
Proof equivalence

if NOT (blue AND NOT red) OR red then...

\[ \neg(p \land \neg q) \lor q \equiv \neg p \lor q \]

\[ \neg(p \land \neg q) \lor q \equiv (\neg p \lor \neg \neg q) \lor q \quad \text{De Morgan’s II} \]

\[ \equiv (\neg p \lor q) \lor q \quad \text{Double negation} \]

\[ \equiv \neg p \lor (q \lor q) \quad \text{Associativity} \]

\[ \equiv \neg p \lor q \quad \text{Idempotent} \]
Another Example

Show that \([p \land (p \rightarrow q)] \rightarrow q\) is a tautology

We use \(\equiv\) to show that \([p \land (p \rightarrow q)] \rightarrow q \equiv T\)

\[
[p \land (p \rightarrow q)] \rightarrow q \\
\equiv [p \land (\neg p \lor q)] \rightarrow q \quad \text{substitution for } \rightarrow \\
\equiv [(p \land \neg p) \lor (p \land q)] \rightarrow q \quad \text{distributive} \\
\equiv [F \lor (p \land q)] \rightarrow q \quad \text{uniqueness} \\
\equiv (p \land q) \rightarrow q \quad \text{identity} \\
\equiv \neg (p \land q) \lor q \quad \text{substitution for } \rightarrow \\
\equiv (\neg p \lor \neg q) \lor q \quad \text{De Morgan’s II} \\
\equiv \neg p \lor (\neg q \lor q) \quad \text{associative} \\
\equiv \neg p \lor T \quad \text{excluded middle} \\
\equiv T \quad \text{domination}
\]
Another Example

- Consider the newspaper headline:
  “Legislature Fails to Override Governor’s Veto of Bill to Cancel Sales Tax Reform”

Did the legislature vote in favor of or against the sales tax reform?
Another Example

• Consider the newspaper headline:
  “Legislature Fails to Override Governor’s Veto of Bill to Cancel Sales Tax Reform”

Did the legislature vote in favor of or against the sales tax reform?

A) The legislature DID vote in favor
B) The legislature DID NOT vote in favor

Let’s take a vote
Another Example

- Consider the newspaper headline:
  “Legislature Fails to Override Governor’s Veto of Bill to Cancel Sales Tax Reform”

Did the legislature vote in favor of or against the sales tax reform?

Let s stand for “sales tax reform”

Unravel one at a time.
  - The bill to cancel sales tax reform is ¬s
  - The governor’s veto of the bill is ¬ ¬s
  - Overriding this means ¬ ¬ ¬s
  - Failing to do so would mean ¬ ¬ ¬ ¬s
  - The two double negations cancel out, leaving just s

Therefore the legislature supports sales tax reform. (Yes, they DID vote in favor)