Announcements

• Exam 3 is on Thursday (No Final Exam)
  • Last Day of Class
  • Covers Chapters 6, 7, 9, 10, and 13

• Allowed to bring in one 8.5 x 11in sheet of paper
  – Hand written notes of anything you would like

• Quiz 10 is today
13.4 Language Recognition

• Previously discussed using finite-state automata as language recognizers

• What sets can be recognized by languages?
  – Regular Sets
    • Introduced by Stephen Kleene and his Kleene Theorem

• Regular Sets
  – Sets built up from the null set \{\}, the empty string \(\lambda\), and singleton strings by taking concatenation, unions, and Kleene closures in arbitrary order
Regular Expressions

• To further define regular sets we need to define *regular expressions*.

• The regular expressions over a set $S$ are defined recursively by:
  – The symbol $\emptyset$ is a regular expression.
  – The symbol $\lambda$ is a regular expression (empty string).
  – The symbol $x$ is a regular expression whenever $x$ is an element of $S$.
  – The symbols $(AB)$, $(A \cup B)$ and $A^*$ are regular expressions whenever $A$ and $B$ are regular expressions.

• Sets represented by regular expressions are called *regular sets*.
Regular Expression Examples

- What are the strings in the regular sets specified by the following:

  - 10*
    - A 1 followed by any number of 0s including no zeros
  - (10)*
    - Any number of copies of 10
  - 0 U 01
    - the string 0 or 01
  - 0 (0 U 1)*
    - Any string beginning with 0
  - (0*1)*
    - Any string not ending in 0
Kleene’s Theorem

- A set is regular if and only if it is recognized by a finite-state automaton (FSA)

- Prove the only if part
  - Every regular set is recognized by a finite-state automaton

- To prove the only if part we need to show the following:
  - Show that \( \{ \} \) is recognized by an FSA (empty set)
  - Show that \( \{ \lambda \} \) is recognized by an FSA (set containing empty string)
  - Show that \( \{ a \} \) is recognized by an FSA whenever \( a \) is a symbol in input language
  - Show that \( AB \) is recognized by an FSA whenever both \( A \) and \( B \) are
  - Show that \( A \cup B \) is recognized by an FSA whenever \( A \) and \( B \) are
  - Show that \( A^* \) is recognized by an FSA whenever \( A \) is
Kleene’s Theorem

- Show that \{ \} is recognized by FSA

- Show that \{\lambda\} is recognized by FSA

- Show that \{a\} is recognized by FSA
Concatenation – Kleene Theorem

- Show that AB is recognized by an FSA whenever A and B are

Transition to final state in \( M_A \) produces a transition to \( s_B \)

Transition from \( s_B \) in \( M_B \) produces a transition from \( s_{AB} = s_A \)

Start state is \( s_{AB} \), \( Sab = Sa \), which if final if \( s_a \) and \( s_B \) are final.
Union – Kleene Theorem

• Show that $A \cup B$ is recognized by an FSA whenever both $A$ and $B$ are

$M_A$ or $M_B$

$s_{AUB}$ is the new start state, which is final if $s_A$ or $s_B$ is final

Final states are the final states of $M_A$ or $M_B$
Kleene Closure – Kleene Theorem

• Show that $A^*$ is recognized by an FSA whenever $A$ is

Transitions from $s_A$ produce $A$ transitions from $s_{A^*}$ and all final states of $M_A$
Practice Problems (Chp 6)

1) How many permutations of the seven letters A, B, C, D, E, F, G do not have vowels on the ends?

2) A class consists of 20 sophomores and 15 freshmen. The club needs to choose four different members to be president, vice president, secretary, and treasurer.

(a) In how many ways is this possible?

(b) In how many ways is this possible if sophomores will be chosen as president and treasurer and freshmen as vice president and secretary?
Solutions Practice Problems (Chp 6)

1) How many permutations of the seven letters A, B, C, D, E, F, G do not have vowels on the ends?
   \[ 5 \times 4 \times 5! = 20 \times 120 = 2,400 \]

2) A class consists of 20 sophomores and 15 freshmen. The club needs to choose four different members to be president, vice president, secretary, and treasurer.

   (a) In how many ways is this possible?
   \[ 35 \times 34 \times 33 \times 32 = 1,256,640 \]

   (b) In how many ways is this possible if sophomores will be chosen as president and treasurer and freshmen as vice president and secretary?
   \[ 20 \times 19 \times 15 \times 14 = 79,800 \]
Practice Problems (Chp 7)

3) What is the probability that a card chosen from an ordinary deck of 52 cards is an ace?

4) Three unbiased coins are tossed. (a) List the elements in the sample space. (b) Find the probability that exactly two heads show.
3) What is the probability that a card chosen from an ordinary deck of 52 cards is an ace? \[ \frac{4}{52} \]

4) Three coins are tossed. (a) List the elements in the sample space.

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

(b) Find the probability that exactly two heads show.

\[ \frac{3}{8} \]
Practice Problem (Chp 7)

- Say I play a game and the probability I win is .7 and I lose is .3. Each time I play the game I win independently from the last times I played. I play the game n times

  - What is the probability of me winning exactly n/10 of the times I play?
  - What is the probability I win at least n/4 times and at most 3/4n?
Practice Problem (Chp 7)

- Say I play a game and the probability I win is .7 and I lose is .3. Each time I play the game I win independently from the last times I played. I play the game n times

  - What is the probability of me winning exactly n/10 of the times I play?
    - \( C(n, n/10)(.7)^{(n/10)}(.3)^{(9/10n)} \)

  - What is the probability I win at least n/4 times and at most 3/4n?
    - \( \sum_{j=n/4}^{3/4n} \binom{n}{j}(.7)^j(.3)^{n-j} \)
Practice Problems Solutions (Chp 9)

In the questions below determine whether the relationship is reflexive, symmetric, antisymmetric, and/or, transitive

5) The relation \( R \) on \( \mathbb{Z} \) where \( aRb \) means \( a^2 = b^2 \).
   Reflexive, Symmetric, Transitive

6) The relation \( R \) on \( \mathbb{Z} \) where \( aRb \) means \( a \neq b \).
   Symmetric

7) The relation \( R \) on the set of all people where \( aRb \) means that \( a \) is at least as tall as \( b \).
   Reflexive and Transitive
Practice Problems (Chp 9)

In the questions below determine whether the relationship is reflexive, symmetric, antisymmetric, and/or, transitive

5) The relation $R$ on $\mathbb{Z}$ where $aRb$ means $a^2 = b^2$.

6) The relation $R$ on $\mathbb{Z}$ where $aRb$ means $a \neq b$.

7) The relation $R$ on the set of all people where $aRb$ means that $a$ is at least as tall as $b$. 
Practice Problems (Chp 13)

8) Find the language recognized by this NFA

9) Find the Kleene closure of $A = \{00\}$
8) Find the language recognized by this NFA

\{1,01^n0,1^{n+1}0 \mid n \geq 0\}

9) Find the Kleene closure of \( A = \{00\} \)

\( A^* = \{ \lambda, 00, 0000, 000000, ... \} \)
Quiz 10