Announcements

• Homework 1 is due now

• Quiz 1 is next Tuesday

• Homework 2 is posted online and due next Thursday

• Read Section 1.6 (Rules of Inference) and 1.7 (Introduction to Proofs) by Tuesday
## Predicates – Multiple Quantifiers

<table>
<thead>
<tr>
<th>Quantifier</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\forall x \forall y , P(x,y)$</td>
<td>$P(x,y)$ true for all $x$, $y$ pairs.</td>
</tr>
<tr>
<td>$\exists x \exists y , P(x,y)$</td>
<td>$P(x,y)$ true for at least one $x$, $y$ pair.</td>
</tr>
<tr>
<td>$\forall x \exists y , P(x,y)$</td>
<td>For every value of $x$ we can find a (possibly different) $y$ so that $P(x,y)$ is true.</td>
</tr>
<tr>
<td>$\exists x \forall y , P(x,y)$</td>
<td>There is at least one $x$ for which $P(x,y)$ is true for every $y$.</td>
</tr>
</tbody>
</table>
Predicates – multiple quantifiers

N(x, y) = “x is sitting by y”

\[ \forall x \forall y \ N(x, y) \quad \text{False} \]

\[ \exists x \exists y \ N(x, y) \quad \text{True} \]

\[ \forall x \exists y \ N(x, y) \quad \text{True} \]

\[ \exists x \forall y \ N(x, y) \quad \text{False} \]

Universe of discourse is all students in this room.

Let “sitting by” be defined as x is sitting within 5 feet of y
A theorem is a statement that can be shown to be true. 

A proof is the means of doing so.
The following statements are true:
If I am Mila, then I am a great swimmer.
I am Mila.

What do we know to be true?
I am a great swimmer!
Proofs - Definitions

• **Argument**
  – A sequence of propositions
  – All but the final proposition are called premises
    • Final proposition called conclusion
  – An argument is valid if the truth of all premises implies the conclusion is true

• **Argument form**
  – Sequence of compound propositions involving proposition variables

```
premise          p
premise          p → q
               _______
conclusion       ∴ q
```
I am Mila.
If I am Mila, then I am a great swimmer.

∴ I am a great swimmer!

\[ p \rightarrow q \]

Inference Rule:
Modus Ponens

Tautology:
\[ (p \land (p \rightarrow q)) \rightarrow q \]

Modus Ponens is Latin for "the way that affirms by affirming"
I am not a great skater.
If I am Erik, then I am a great skater.

∴ I am not Erik!

Tautology:
\[(\neg q \land (p \rightarrow q)) \rightarrow \neg p\]

Inference Rule:
Modus Tollens

Modus Tollens is Latin for "the way that denies by denying"
I am not a great skater.

∴ I am not a great skater or I am a monkey.

\[ p \rightarrow (p \lor q) \]

Inference Rule:
Addition or Weakening
I am not a great skater and you are sleepy.

∴ you are sleepy.

\[
\begin{align*}
p \land q & \\
\hline
\therefore p & \\
\end{align*}
\]

**Tautology:**
\[
(p \land q) \implies p
\]

**Inference Rule:** Simplification

---

**Proofs - Simplification**
Proofs - Disjunctive Syllogism

I am a great eater or I am a great skater.
I am not a great skater.

∴ I am a great eater!

\[
p \lor q,
\neg q
\]

\[
\therefore p
\]

Tautology:

\[
((p \lor q) \land \neg q) \rightarrow p
\]

Inference Rule:
Disjunctive Syllogism
If you are an athlete, you are always hungry.
If you are always hungry, you have a snickers in your backpack.

∴ If you are an athlete, you have a snickers in your backpack.

$p \rightarrow q$
$q \rightarrow r$

\[ ((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r) \]

Inference Rule:
Hypothetical Syllogism
I am a great athlete
I am always hungry

∴ I am a great athlete and always hungry

\[
p \land q
\]

Tautology:

\[
[(p) \land (q)] \rightarrow (p \land q)
\]

Inference Rule:

Conjunction
Proofs - fallacies

Rules of inference, appropriately applied give valid arguments.

Mistakes in applying rules of inference are called fallacies.
If I am Bonnie Blair, then I skate fast.
I skate fast!

\[ \therefore \text{I am Bonnie Blair} \]

I’m Eric Heiden

If you don’t give me $10, I bite your ear.
I bite your ear!

\[ \therefore \text{You didn’t give me $10.} \]

I’m just mean.

Affirming the conclusion.

\[ ((p \rightarrow q) \land q) \rightarrow p \]

Not a tautology.
Valid Argument or Fallacy?

If it rains then it is cloudy.
It does not rain.

\[ \therefore \text{It is not cloudy} \]

If it is a car, then it has 4 wheels.
It is not a car.

\[ \therefore \text{It doesn’t have 4 wheels.} \]

Denying the hypothesis.

\[ ((p \rightarrow q) \land \neg p) \rightarrow \neg q \]

ATV

Not a tautology.

January!