Announcements

• Homework 3 is due today
  – Turn it in now

• Exam 1 is in one week
  – Thursday Feb 12th
Exam 1 Review

• Exam 1 covers all of Chapter 1

• Questions will be a mix of short answer, multiple choice

• Be prepared to prove propositions using
  – Direct Proofs
  – Contradictions
  – Contrapositions

• Each step in a proof needs to clearly follow from the previous
  – Modus Tollens
  – Modus Ponens
  – definition of an odd
  – Simple algebra
  – Etc...
Exam 1 Review

• Additional terms to be familiar with
  – Natural Numbers
    • \{0, 1, 2, 3...\}
  – Integers
    • \{..., -2, -1, 0, 1, 2, ...\}
  – Rational Numbers
    • \{p/q \text{ where } p \text{ and } q \text{ are integers and } q \neq 0\}
  – Real Numbers
    • Non imaginary numbers
  – Prime Numbers
    • Integers > 1 with no positive divisors besides 1 and itself \{2, 3, 5, 7, 11, and many more\}
  – Composite Numbers
    • Positive integers > 1 that are not prime

• Proof Fallacies
  – Affirming the conclusion
    • If you do every problem in this book, then you will learn Discrete Mathematics
    • You learned Discrete Mathematics
    • Therefore you did every problem in the book
  – Denying the hypothesis
    • \( p \rightarrow q, \neg p, \text{ therefore } \neg q\)
Example (#1)

• Break up into groups of 3 or 4

• Determine whether this argument is valid:

• Lynn works part time or full time.
• If Lynn does not play on the team, then she does not work part time.
• If Lynn plays on the team, she is busy.
• Lynn does not work full time.
• Therefore, Lynn is busy.
Examples (#1 Solution)

- Lynn works part time
- Lynn works full time
- Lynn plays on the team
- Lynn is busy

1) \( p \lor f \)  
   premise
2) \( \neg t \rightarrow \neg p \)  
   premise
3) \( t \rightarrow b \)  
   premise
4) \( \neg f \)  
   premise
5) \( p \)  
   Disjunctive syllogism 1 and 4
6) \( p \rightarrow t \)  
   Contrapositive of 2
7) \( t \)  
   Modus ponens on 5 and 6
8) \( b \)  
   Modus ponens on 7 and 3

Therefore, Lynn is busy.
Prove or disprove that the square of every even integer ends in 0, 4, or 6.
More Examples (#2 solutions)

Prove or disprove that the square of every even integer ends in 0, 4, or 6.

Every even integer \( n \) can be written as \( n = 10k + r \) where \( r = 0, 2, 4, 6, 8 \). (For example, 34 = 10 \times 3 + 4 and 6 = 6 \times 0 + 6.)

Examine each of these five cases separately:

\[ n = 10k + 0: n^2 = 100k^2, \text{ which ends in 0 (it is a multiple of 10)} \]

\[ n = 10k + 2: n^2 = 100k^2 + 40k + 4 = 10(10k^2 + 4k) + 4 \text{ and hence ends in 4} \]

\[ n = 10k + 4: n^2 = 100k^2 + 80k + 16 = 10(10k^2 + 8k + 1) + 6 \text{ and hence ends in 6} \]

\[ n = 10k + 6: n^2 = 100k^2 + 120k + 36 = 10(10k^2 + 12k + 3) + 6 \text{ and hence ends in 6} \]

\[ n = 10k + 8: n^2 = 100k^2 + 160k + 64 = 10(10k^2 + 16k + 6) + 4 \text{ and hence ends in 4}. \]
In the questions below suppose the variable $x$ represents students and $y$ represents courses, and:

$F(x)$: $x$ is a freshman
$A(x)$: $x$ is a part-time student
$T(x,y)$: $x$ is taking $y$

Write the statement in good English without using variables in your answers.

3. $F$(Mikko).

4. $\neg\exists y\ T$(Joe,$y$).

5. $\exists x\ (A(x) \land \neg F(x))$. 
More Examples (#3,#4, and #5 Solutions)

In the questions below suppose the variable $x$ represents students and $y$ represents courses, and:

$F(x): x$ is a freshman
$A(x): x$ is a part-time student
$T(x,y): x$ is taking $y$

Write the statement in good English without using variables in your answers.

3. $F$(Mikko). Miko is a freshman

4. $\neg \exists y \ T$(Joe,$y$). Joe is not taking any course

5. $\exists x (A(x) \land \neg F(x))$. Some part-time students are not freshman
More Examples (#6 & #7)

• 6) Determine whether $p \rightarrow (q \rightarrow r)$

and $p \rightarrow (q \land r)$ are equivalent

• 7) Prove that $(\neg p \land (p \lor q)) \rightarrow q$ is a tautology using propositional equivalence and the laws of logic
More Examples (#6 solutions)

6) Determine whether $p \rightarrow (q \rightarrow r)$ and $p \rightarrow (q \land r)$ are equivalent.

Ans: Not equivalent. Let $q$ be false and $p$ and $r$ be true.
More Examples (#7 solutions)

Prove that \((\neg p \land (p \lor q)) \rightarrow q\) is a tautology using propositional equivalence and the laws of logic

Ans:

\[
(\neg p \land (p \lor q)) \rightarrow q \\
\equiv ((\neg p \land p) \lor (\neg p \land q)) \rightarrow q \quad \text{(Distributivity)} \\
\equiv (F \lor (\neg p \land q)) \rightarrow q \quad \text{(Uniqueness)} \\
\equiv (\neg p \land q) \rightarrow q \quad \text{(Domination)} \\
\equiv \neg (\neg p \land q) \lor q \quad \text{(Defn } \rightarrow) \\
\equiv (\neg p \lor \neg q) \lor q \quad \text{(De Morgan’s)} \\
\equiv (p \lor \neg q) \lor q \quad \text{(Associativity)} \\
\equiv p \lor (\neg q \lor q) \quad \text{(Defn } \lor) \\
\equiv p \lor (T) \quad \text{(Defn } T) \\
\equiv T
\]
More Examples (#8 & #9)

8) Find a proposition with three variables \( p, q, \) and \( r \) that is true when exactly one of the three variables is true, and false otherwise.

9) Find a proposition with three variables \( p, q, \) and \( r \) that is never true.
More Examples (#8 & #9 solutions)

8) Find a proposition with three variables $p$, $q$, and $r$ that is true when exactly one of the three variables is true, and false otherwise.

Ans: $(p \land \neg q \land \neg r) \lor (\neg p \land q \land \neg r) \lor (\neg p \land \neg q \land r)$

9) Find a proposition with three variables $p$, $q$, and $r$ that is never true.

Ans: $(p \land \neg p) \lor (q \land \neg q) \lor (r \land \neg r)$
Group Exercise

- A corporate businessman has two cubes on his office desk. Every day he arranges both cubes so that the front faces show the current day of the month.
- What numbers are on the faces of the cubes to allow this?

- Note: You can't represent the day "7" with a single cube with a side that says 7 on it. You have to use both cubes all the time. So the 7th day would be "07".