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Fairness: How to Measure It Quantitatively?

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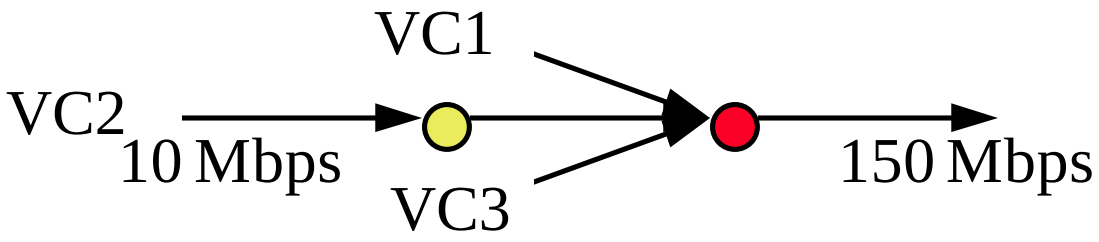
- Index of fairness
- Why is it better than others?

References:

[1] R. Jain, W. Hawe, D. Chiu, "A Quantitative measure of fairness and discrimination for resource allocation in Shared Computer Systems," DEC-TR-301, September 26, 1984.

[2] Raj Jain, "The Art of Computer Systems Performance Analysis," Wiley 1991

Fairness



- Simple Definition: Equal share of bottleneck
  Problem: Some VC's may be bottlenecked elsewhere
- Next Definition: Optimal Allocation or Equal fraction of optimal allocation
- Example: A scheme gives 100, 4, 10.5 Mbps when the optimal is 100, 40, 15 Mbps
  How fair is it? 67% ? 90%?

Proposal

- Actual allocation: $(A_1, A_2, ..., A_n)$
- Use any criterion (e.g., max-min optimality) to find the optimal allocation $(O_1, O_2, ..., O_n)$
- Relative allocation: $x_i = A_i/O_i$

$$\text{Fairness} = \frac{(\Sigma x_i)^2}{n\Sigma x_i^2}$$

Example: 100/100, 4/40, 10.5/15 $\Rightarrow$ 1, 0.1, 0.9

$$\text{Fairness} = \frac{(1+0.1+0.9)^2}{3(1^2+0.1^2+0.9^2)} = \frac{2^2}{3(1+0.01+0.81)} = 0.73$$

Other Proposals

- Find the variance, standard deviation, or coefficient of variation

  Mean $\mu = (1+0.1+0.9)/3 = 0.67$

  Variance $\sigma^2 = (1/n)\Sigma(x_i-\mu)^2 = 0.16$

  Standard deviation $\sigma = 0.4$

  Coefficient of variation = $\sigma/\mu = 0.597$

- Find the distance from the optimal

$$\text{Fairness} = \frac{[\Sigma (A_i-O_i)^2]^{1/2}}{[\Sigma O_i^2]^{1/2}} = \frac{[0^2+0.9^2+0.1^2]^{1/2}}{[1^2+1^2+1^2]^{1/2}} = 0.52$$

- Min/Max = 0.1/1 = 0.1

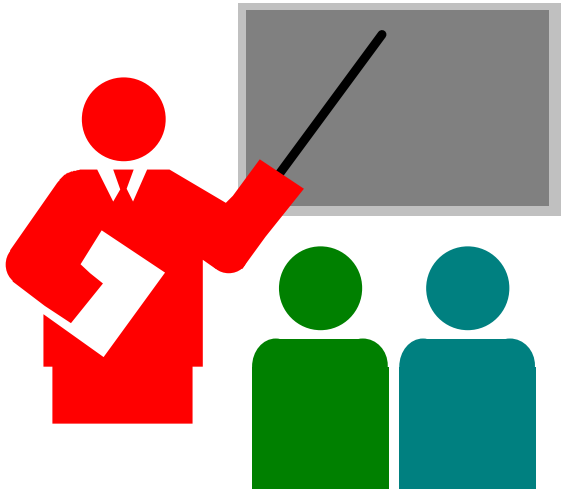
Fairness Index: Properties

- Applicable for any number of VCs, even n=2
  Strictly speaking, variance not defined for small n.
- Scale independent.
  Variance (Throughput) = 10 $Mbps^2$ = 10^7 $kbps^2$
  Standard deviation (Throughput) = 10 Mbps = 10^4 kbps
- Bounded between 0 and 1 or 0 and 100%
  Variance, standard deviation, and Relative distance are not bounded.
- Direct relationship: Higher index⇒More Fair
  Higher variance ⇒ Less fair
- Continuous. Min/max is not continuous.

Fairness Index: Properties

- Intuitive:
  - For (1, 0, 1) Index = 2/3
  - For $x_i = 1$, i=1,2,3,...,k
    $= 0$ otherwise
    Index = k/n
  - If 80% of the users are treated fairly and 20% are starved, index = 80%

Summary



- The following text be added to the baseline text.

The fairness will be quantified using the following formula :

$$\text{Fairness} = \frac{(\Sigma x_i)^2}{n\Sigma x_i^2}$$

where x_i = ratio of actual throughput/optimal throughput.