

Analytical Modeling of Beyond Visual Range Air Combat

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Abstract

The paper reviews fundamental beyond-visual-range air-combat analytical models and enhances these models. In particular, the fundamental models reviewed are sequential missile exchanges. A one-versus-one model is developed to handle the case when both sides are destroyed in a single exchange. The paper then develops an analytical model that takes into account engagement geometry, detection range, aircraft speed, missile speed, and missile seeker type for a single missile exchange. This type of model can be used to predict the order of missile exchanges.

Keywords: Air Combat, Beyond Visual Range, Analytical Modeling

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1. Introduction

There is an extensive history of applying mathematics to military science. During World War II the profession operations research was created to use mathematics to address military problems. During the same time period, warfare was changing rapidly due to new technology such as aircraft. A new branch of the United States Military was founded, the Air Force, to best exploit this new domain of warfare. Since its inception, the Air Force has relied on science and mathematics to determine investments in technology and form its doctrine. The quintessential model of combat was developed by Lanchester while contemplating the impact aircraft would have on warfare (1). During the cold war simple analytical models were developed and used to understand air-to-air combat in the jet-age. The Rand Corporation played a key role in the development of these models.

Today, simulation is the main tool used to study complex military problems. Performance analysis of various military systems and systems-of-systems is often the subject. There are many reasons for the predominance of simulation in the study of the performance of military systems. The reasons are not unique to the study of military systems, for example Dr Jain's book on computer systems performance analysis outlines the strengths of computer simulation (2). Among them are moderate fidelity, moderate abstraction, and the ability to study systems that do not yet exist. All of these reasons make it more saleable to decision makers. Nevertheless, simulation can be a time intensive and still requires verification from other methods such as measurement, analytical models, or expert opinion. Analytical models, while lacking detail, are simple and fast. They are useful as screening tools and understanding more complex simulations. They can also be used in large computer simulations when fast run times are necessary or when a simulation requires modeling a phenomenon but it is not the subject of the study. In order to be accepted by decision makers, the assumptions that are made in analytical models must not impair the purpose of the analysis.

Historic analytical models to study air combat for beyond-visual-range (BVR) missile exchanges, while having there place, do not take into account factors important to current-day aircraft. At the very least a useful model should include the geometry of the engagement, detections, speeds, and types of missiles used. The purpose of this paper is two-fold. The first objective is to survey several analytical models of air-to-air BVR missile exchanges. The second objective is to expand on these models to create a useful model to analyze current and future aircraft performance.

2. Fundamental Analytical Models

2.1 Undefended Target - One-Sided Engagements

Basic missile exchange models start with the probability of a single missile destroying a target (3). This quantity is known as the single shot probability of kill and is denoted by PKSS. Sequential exchanges of missiles between forces can be modeled to determine either the expected number of survivors or kills. For example, for a one-sided exchange between a fighter aircraft carrying air-to-air missiles (AAM) and a bomber not carrying AAM, the equations are particularly simple. The probability of the bomber surviving the fighter attack is determined by equation 1 and the probability of the fighter destroying the target is given by equation 2.

$$S(k) = (1 - p)^k \quad (1)$$

$$P(k) = 1 - S(k) \quad (2)$$

k - number of shots

p - probability of a single shot destroying the target

S(k) - probability of target surviving salvo of shots

P(k) - probability of target destroyed by salvo of shots

Figure 1 illustrates equation 1 and 2 for various values of p and k. Although, this model is extremely simple, it is appropriate for modeling the attack of a slow-moving undefended target.

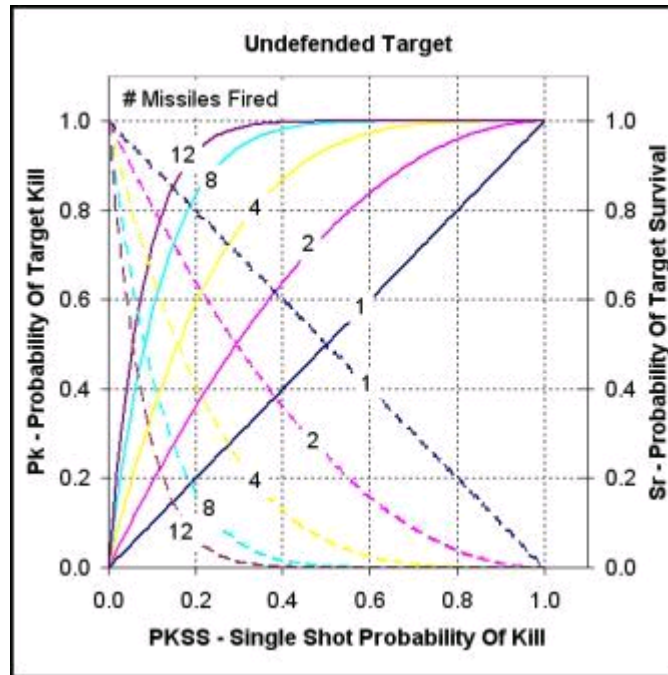


Figure 1

2.2 One Versus One Sequential Engagements

A similar model can be developed for an exchange of missiles between two aircraft (3). This is known as a 1v1 engagement where v stands for versus. As is convention, one side will be known as blue and the other red. For example, we will assume blue fires first. Blue fires first and the probability of red being destroyed is p_B and the probability that red survives is $1-p_B$. Red responds and the probability of blue being destroyed is now $p_R(1-p_B)$ and probability of blue surviving is $1-p_R(1-p_B)$. The sequential exchange continues until both sides are out of weapons. When blue shoots first, equation 3 and 4 are used to represent the probability of red being destroyed by blue after n blue weapons are fired and the probability of blue being destroyed by n red weapons, respectively. Conversely if red fires first, equation 5 and 6 are used. Figure 2a illustrates equations 3 and 4 for various values of p_B and p_R , and figure 2b illustrates equation 5 and 6.

$$P_{BR}(n) = \frac{p_B [1 - (1-p_B)^n (1-p_R)^n]}{1 - (1-p_B)(1-p_R)} \quad (3)$$

$$P_{BB}(n) = \frac{p_R(1-p_B) [1 - (1-p_B)^n (1-p_R)^n]}{1 - (1-p_B)(1-p_R)} \quad (4)$$

$$P_{RR}(n) = \frac{p_R(1-p_B) [1 - (1-p_B)^n (1-p_R)^n]}{1 - (1-p_B)(1-p_R)} \quad (5)$$

$$P_{RB}(n) = \frac{p_R [1 - (1-p_B)^n (1-p_R)^n]}{1 - (1-p_B)(1-p_R)} \quad (6)$$

$PBR(n)$ - when blue shoots first, probability red is destroyed
 $PBB(n)$ - when blue shoots first, probability blue is destroyed
 $PRR(n)$ - when red shoots first, probability red is destroyed

PRB(n) - when red shoots first, probability blue is destroyed
 pB - blue probability of single shot destroying red
 pR - red probability of single shot destroying blue
 n - number of missiles fired by red/blue

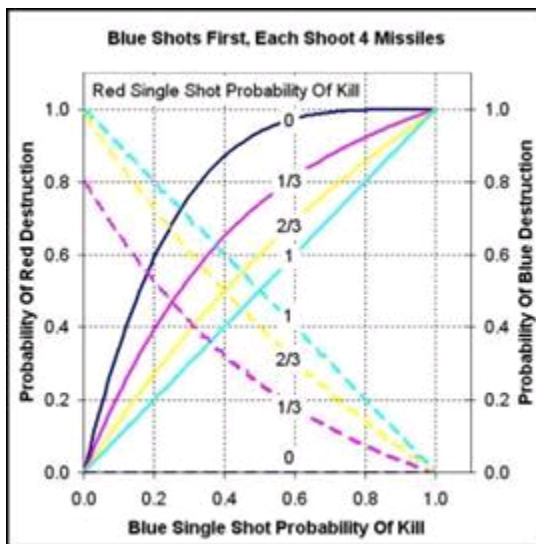


Figure 2a

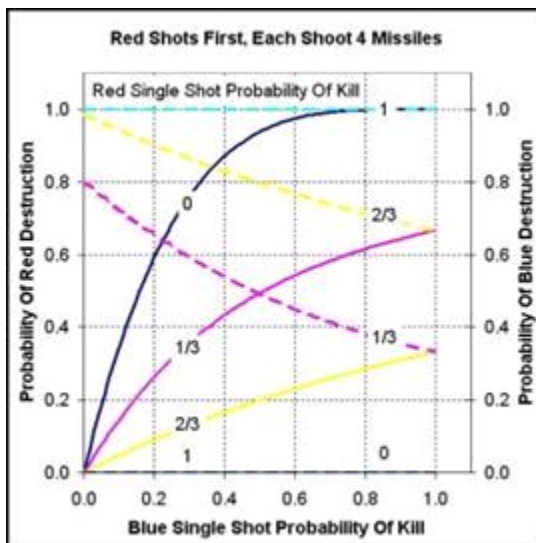


Figure 2b

The obvious conclusion from figure 2a and 2b is shooting first is very important, even more important than weapon PKSS.

2.3 Many Versus Many One-Sided Engagements

The model in section 2.1 can be extended beyond 1v1 engagements to 2v2, 2v4, or NvM in general (4, 5). The convention for the nomenclature is the number of blue aircraft is listed first, then v, and finally the number of red aircraft.

Three cases have been developed in the reference 4 for dealing with target sorting. In one extreme, each attacker randomly picks its targets. In this case, some targets may be double targeted and weapons may be wasted. This case may represent when there is poor command-and-control and/or poor communication

between the individual attackers. The other extreme is perfect target sorting where all attackers uniformly distribute weapons over the targets. This case represents when the attackers and/or their controllers have good communication and the ability to effectively sort targets. The third case is an extension of the second where the attackers and/or their controllers have the ability to make a good kill assessment allowing them to focus new salvos only on surviving targets.

This paper will focus on the simple extension of the 1v1 engagement to an NvN engagement. It is important to demonstrate that 1v1 results can be extended to many-on-many, but the details can be worked out in latter. In this situation $N = M$, and each attacker is assigned a single target. In addition the attacker expends all its missiles on its assigned target. Equations 7-10 can be used to determine the average number of survivors after the engagement is complete.

$$N = MJ \quad (7)$$

$$Sr = (1 - p_J)^{N/T} \quad (8)$$

$$p_J = 1 - (1 - p)^J \quad (9)$$

$$E_s = T \cdot Sr \quad (10)$$

J - number of weapons per attacker

M - number of attackers

N - total number of weapons

T - number of targets

p_J - probability of kill for J shots

p - probability of a single shot destroying the target

Sr - average survivability of each target

E_s - expected number of surviving targets

Since $M = T$, equations 7-10 can be simplified to equation 11.

$$Sr = (1 - p)^{J \cdot J} \quad (11)$$

3. Extensions To Fundamental Analytical Models

3.1 One Versus One Sequential Engagement - Mutual Kills

Although the chance of both sides getting destroyed simultaneously may seem remote, with missiles that are actively guided in the terminal phase it can occur. This type of missile only needs to be guided by the attacker to within a certain range of the target, and then the attacker may disengage from the fight. This is advantage compared to the less sophisticated semi-active guided missile that must be guided all the way to the target (4). The author was not able to find the derivation of a 1v1 sequential engagement in which both sides' shots arrive simultaneously resulting in a mutual kill. Therefore equations are derived for this situation. The missiles in this case no longer arrive sequentially, but each side's volley arrives simultaneously. With each volley there are four potential outcomes: both sides are destroyed, red is destroyed and blue survives, red survives and blue is destroyed, or both sides survive. For example, equation 12 is the probability red is destroyed in the first exchange. It is the sum of the probability both sides are destroyed, and red is destroyed and blue survives.

$$P_{1R} = p_B \cdot p_R + p_B(1 - p_R) \quad (12)$$

Similarly, equation 13 represents the probability red is destroyed in the second volley. It is the probability that red and blue survived the first volley multiplied by the two cases where red is destroyed.

$$P_{2R} = (1 - p_B)(1 - p_R)[p_B \cdot p_R + p_B(1 - p_R)] \quad (13)$$

The natural result of this procedure is a general equation for n shots. Equation 14 and 15 represent the probability of destruction after n shots for red and blue, respectively.

$$P_R = \frac{p_B(1 - (1 - p_B)^n(1 - p_R)^n)}{1 - (1 - p_B)(1 - p_R)} \quad (14)$$

$$P_B = \frac{p_R(1 - (1 - p_B)^n(1 - p_R)^n)}{1 - (1 - p_B)(1 - p_R)} \quad (15)$$

A careful reader will note the resulting equation 12 is the same as equation 3. Similarly equation 13 is the same as equation 6.

Figure 3 illustrates equation 12 and 13 for various values of p_B and p_R . As might be expected, the results fall somewhere between blue shoots first and red shoots first.

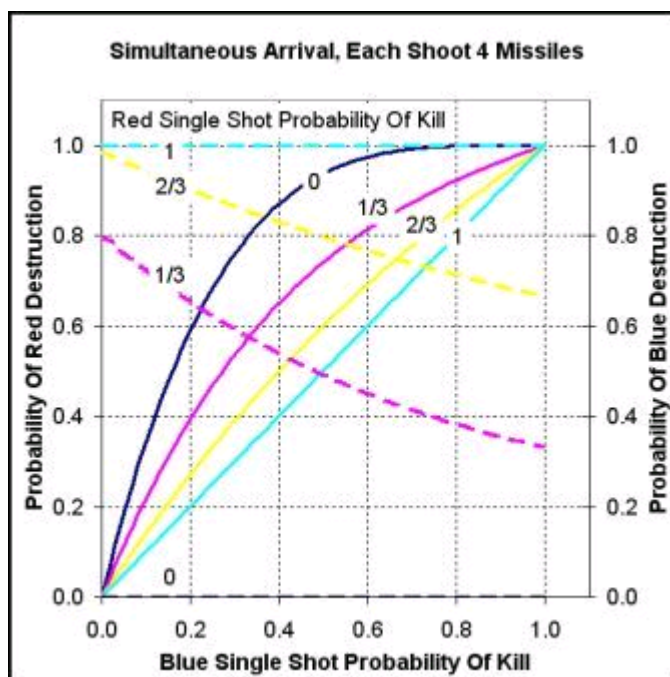


Figure 3

3.2 Including Engagement Geometry, Detection, Speed, And Different Weapon Technologies

The analytical models presented so far lack several important factors to modern air combat. For example there is no consideration of range and speed. They can not answer whether it is feasible for each side to launch some number of weapons at each other, or predict which side might get the first shot based on weapon

system characteristics. To answer these deficiencies it is necessary to introduce ranges and speeds. Reference 5 develops an analytical model that includes these important factors. The model that follows, builds on that developed in reference 5 for our purposes.

We make the simplifying assumption that the engagement takes place in a plane. We develop the situation from the perspective that blue is the aggressor, but the model can be developed for either side as the aggressor. Blue desires to intercept red. Assuming a solution is possible, a collision course provides the minimum time to intercept. Frame T_0 of figure 4 illustrates this situation in two dimensions. The line that intersects both aircraft is known as the line of sight or LOS. The heading of blue relative to the LOS is computed using equation 16 which is an application of the law of sine. Equation 17 can be used to find the closure rate which is an application of the law of cosine.

$$\beta = \arcsin\left(\frac{V_R}{V_B} \sin(\rho)\right) \quad \text{for } \left|\frac{V_R}{V_B} \sin(\rho)\right| \leq 1 \quad (16)$$

$$V_C = V_B^2 + V_R^2 - 2 \cdot V_B \cdot V_R \cdot \cos(\alpha) \quad (17)$$

ρ - red heading relative to LOS

V_R - red speed

V_B - blue speed

β - blue heading relative to LOS

V_C - closure rate

α - collision angle

Because the range of \sin is $[-1,1]$, \arcsin will be undefined if $V_R/V_B \cdot \sin(\rho)$ is outside this range. If $V_R/V_B < 1$ than this condition is met. If $V_R/V_B \geq 1$, but $\rho \leq \arcsin(V_B/V_R)$ then the condition is met. These are necessary conditions for a collision intercept to exist, but not sufficient. It is necessary and sufficient for a collision intercept if β exists and $V_C > 0$.

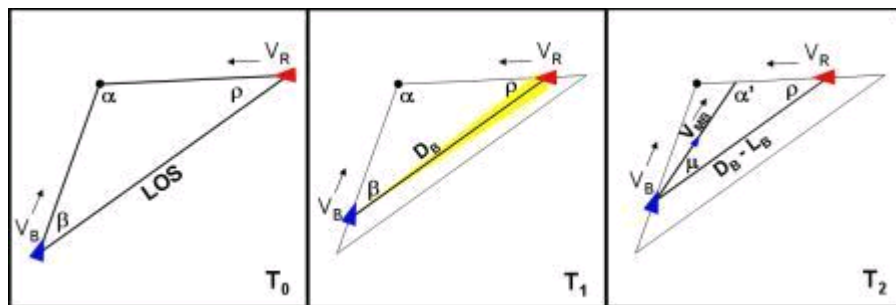


Figure 4

The probability of blue detecting red is assumed to be normally distributed with range. D_B is a random variable and represents the range at which blue detects red. There is also a normally distributed time delay before blue launches its missile at red after detection occurs. Equation 18 defines D_B and T_B . The time delay is represented by random variable T_B . The decrease in distance between blue and red during the time T_B is, of course, a function of T_B and therefore a random variable. It is represented by L_B and given by equation 19. Therefore the range at which blue launches its first missile at red is $D_B - L_B$.

$$\begin{aligned} D_B &\sim N[d_B, \sigma_{DB}] \\ T_B &\sim N[t_B, \sigma_{TB}] \end{aligned} \quad (18)$$

$$\begin{aligned} L_B &= V_C \cdot T_B \\ l_B &= V_C \cdot t_B \\ \sigma_{LB} &= V_C \cdot \sigma_{TB} \end{aligned} \quad (19)$$

A desired characteristic of the model is the blue aircraft must detect the red aircraft before launching its missile. The normal distribution, assumed for simplicity, can violate this condition. Care must be taken to choose a t_B and σ_{TB} such that the probability of launching before detection is very small i.e. t_B should be $3\sigma_{TB}$ away from 0.

The line the blue missile flies is different than the blue aircraft because the blue missile is traveling at a different speed. For simplicity it is assumed the speed at which the missile travels is the summation of the aircraft speed and a base missile speed. The line the missile follows is determined by equations 20 - 22.

$$\mu_B = \arcsin \left(\frac{V_R}{(V_{MB} + V_B)} \sin(\rho) \right) \quad (20)$$

$$V_{CMB}^2 = (V_{MB} + V_B)^2 + V_R^2 - 2 \cdot (V_{MB} + V_B) \cdot V_R \cdot \cos(\alpha_{MB}) \quad (21)$$

$$R_{MB} = \left(\frac{\sin(\rho)}{\sin(\alpha_{MB})} \right) \cdot (D_B - L_B) \quad (22)$$

V_{MB} - blue missile speed without including speed imparted by launching aircraft

μ_B - blue missile heading relative to line of sight

α_{MB} - blue missile collision angle with red aircraft

V_{CMB} - closure rate of blue missile and red aircraft

R_{MB} - distance blue missile flies to get to red aircraft

The distance separating blue and red once the blue missile destroys red is $D_B - L_B - M_B$. Equation 23 is used to calculate M_B . The quantity R_{MB} / V_{CMB} is the time of flight (TOF) of the missile. Equation 24 is found by substituting for M_B in $D_B - L_B - M_B$

$$M_B = \frac{R_{MB}}{V_{CMB}} \cdot V_C \quad (23)$$

$$D_B - L_B - M_B = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}} \right) \cdot (D_B - L_B) \quad (24)$$

Radar seekers are most common for BVR missiles. There are two types of radar seekers used: semi-active radar (SAR) and active radar (AR). For air combat, AR seekers have been used on newer missiles. SAR guided missiles must be supported until impact with the target. AR guided missiles only have to be supported until the missile is within a certain range of the target (4). The advantages of this can be seen in the application of the above developed model.

As in reference 5, the probability of blue destroying red before red reaches launch range can be computed. This computation is for a SAR guided missile. This computation and comparison is a good way to validate the analytical model. The condition that must be met for blue to destroy red before red reaches launch range is

given by equation 25. Z_B or "blue range advantage" is given by equation 26. The probability that $Z_B > 0$ is the probability that blue destroys red before red reaches launch range.

$$\left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}}\right) \cdot (D_B - L_B) > D_R - L_R \quad (25)$$

$$Z_B = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}}\right) \cdot (D_B - L_B) - (D_R - L_R) > 0 \quad (26)$$

Because all random variables were assumed normally distributed the mean and variance of Z_B can be easily computed and are given by equation 27.

$$\mu_{ZB} = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}}\right) \cdot (d_B - l_B) - (d_R - l_R)$$

$$\sigma_{ZB}^2 = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}}\right)^2 \cdot (\sigma_{DB}^2 + \sigma_{LB}^2) + (\sigma_{DR}^2 + \sigma_{LR}^2) \quad (27)$$

For an example, a situation is constructed for which blue and red are equally matched. Generic values were chosen for input to the analytical model and can be seen on the left half of figure 5. The results can be seen on the right half of figure 5. Blue has a 12% probability of destroying red before red reaches launch range. This compares very well with reference 5, although not exactly because the model has been enhanced.

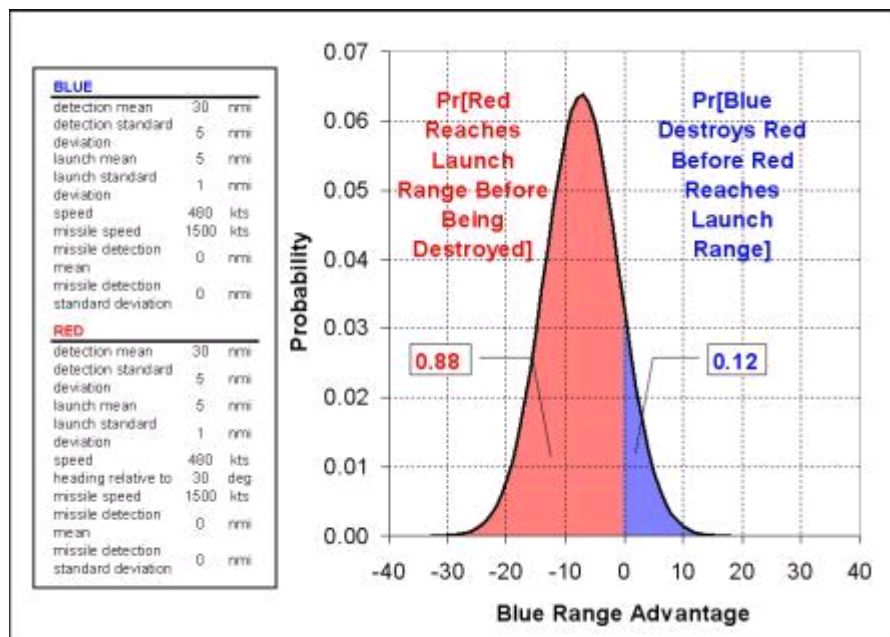


Figure 5

A similar analysis can be done with a slight enhancement to the above model for AR missiles. In this case there is an additional random variable, D_{MB} , to account for the range the missile seeker acquires the target. The extra random variable is added to equation 26 to make equation 28. The mean and standard deviation for equation 28 are given by equation 29.

$$Z_B = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}} \right) \cdot (D_B - L_B + D_{MB}) - (D_R - L_R) > 0$$

$$D_{MB} \sim N[d_{MB}, \sigma_{DMB}] \quad (28)$$

$$\mu_{ZB} = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}} \right) \cdot (d_B - l_B + d_{MB}) - (d_R - l_R)$$

$$\sigma_{ZB}^2 = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}} \right)^2 \cdot (\sigma_{DB}^2 + \sigma_{LB}^2 + \sigma_{DMB}^2) + (\sigma_{DR}^2 + \sigma_{LR}^2) \quad (29)$$

The same example as before is re-worked with the AR missile model. The results can be seen in figure 6. As would be expected, blue fairs better. Blue has a 28% probability of destroying red before red reaches launch range.

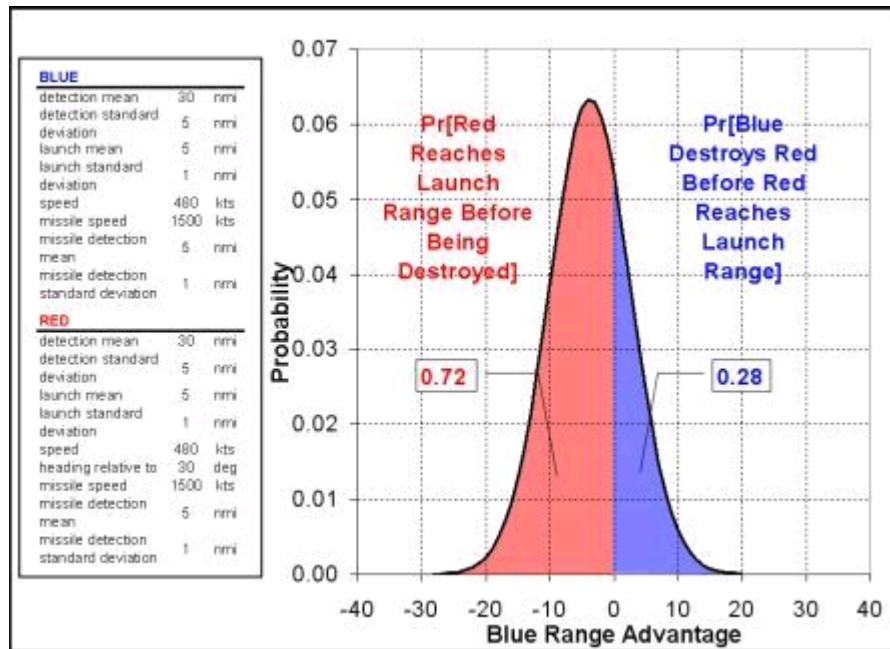


Figure 6

The above model can be further enhanced to represent a single missile dual. A blue "win" happens when the blue missile destroys red before the red missile seeker acquires. Conversely, a red "win" happens when the red missile destroys blue before the blue missile seeker acquires. A third outcome, a mutual kill, is possible. It happens when either side's missile acquires before the launcher is destroyed. The condition when blue wins is represented by equation 30 and the condition when red wins is represented by equation 31.

$$Z_B = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}} \right) \cdot (D_B - L_B) - \left(1 - \frac{\sin(\beta)}{\sin(\alpha_{MR})} \cdot \frac{V_C}{V_{CMR}} \right) \cdot (D_R - L_R + D_{MR}) < 0 \quad (30)$$

$$Z_R = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}} \right) \cdot (D_B - L_B + D_{MB}) - \left(1 - \frac{\sin(\beta)}{\sin(\alpha_{MR})} \cdot \frac{V_C}{V_{CMR}} \right) \cdot (D_R - L_R) < 0 \quad (31)$$

The mean and variance for equation 30 and equation 31 are computed in equation 32 and equation 33, respectively. The probability of a mutual kill is given by equation 34.

$$\mu_{ZB} = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}}\right) \cdot (d_B - l_B) - \left(1 - \frac{\sin(\beta)}{\sin(\alpha_{MR})} \cdot \frac{V_C}{V_{CMR}}\right) \cdot (d_R - l_R + d_{MR})$$

$$\sigma_{ZB}^2 = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}}\right)^2 \cdot (\sigma_{DB}^2 + \sigma_{LB}^2) + \left(1 - \frac{\sin(\beta)}{\sin(\alpha_{MR})} \cdot \frac{V_C}{V_{CMR}}\right)^2 \cdot (\sigma_{DR}^2 + \sigma_{LR}^2 + \sigma_{DMR}^2) \quad (32)$$

$$\mu_{ZR} = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}}\right) \cdot (d_B - l_B + d_{MB}) - \left(1 - \frac{\sin(\beta)}{\sin(\alpha_{MR})} \cdot \frac{V_C}{V_{CMR}}\right) \cdot (d_R - l_R)$$

$$\sigma_{ZR}^2 = \left(1 - \frac{\sin(\rho)}{\sin(\alpha_{MB})} \cdot \frac{V_C}{V_{CMB}}\right)^2 \cdot (\sigma_{DB}^2 + \sigma_{LB}^2 + \sigma_{DMB}^2) + \left(1 - \frac{\sin(\beta)}{\sin(\alpha_{MR})} \cdot \frac{V_C}{V_{CMR}}\right)^2 \cdot (\sigma_{DR}^2 + \sigma_{LR}^2) \quad (33)$$

$$P_{MK} = 1 - P(Z_B < 0) - P(Z_R < 0) \quad (34)$$

Again, a small example is presented to illustrate the analytical model. As before, blue and red have the same inputs and therefore are evenly matched. Figure 7 shows the results. As might be expected for an evenly matched pair, blue wins a quarter of the time, red wins a quarter of the time, and half the time the dual result in a mutual kill.

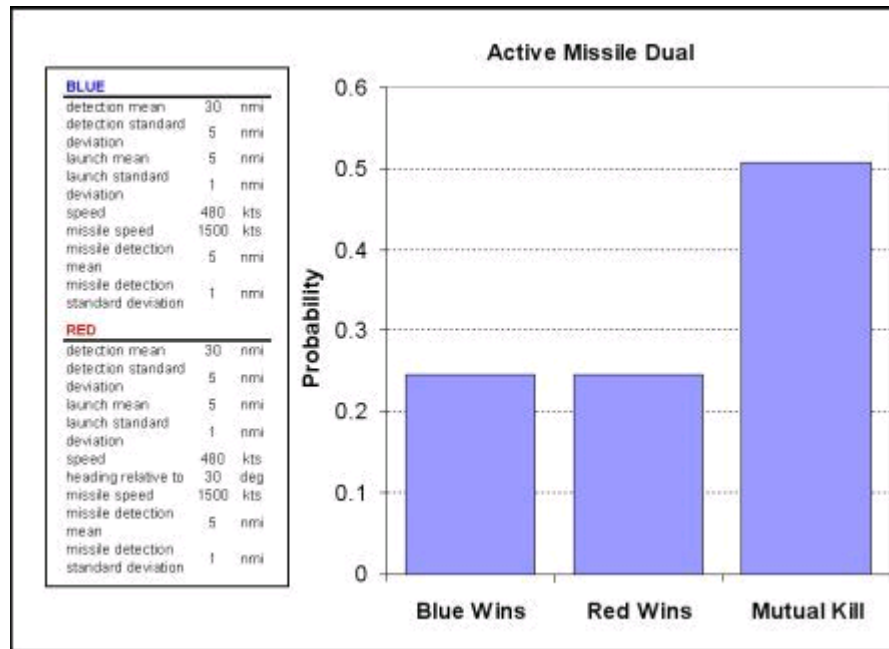


Figure 7

4. Conclusion

The end goal of this investigation is a comprehensive analytical model or set of analytical models useful for BVR air-combat. To this end, a review of sequential engagement models is conducted and these models as well as others are expanded to make them more useful. In particular analytical models are developed to represent the special case of mutual kills, and a model is developed that includes several important factors such as engagement geometry, detection range, missile speed, and missile type for a single missile engagement.

Sequential models are fundamental because more complex models usually result in a PKSS exchange. If this is the case, regardless of the complexity of the model, the results can be reproduced if the exchange order is known e.g. a tree diagram can be used to determine probabilities of survival, etc (4). Unfortunately, the exchange is rarely "textbook" back and forth. Models that take into account engagement geometries and speed are necessary to predict the exchange order and how many weapons are exchanged per time period.

The utility of a "toolbox" of analytical models would be the ability to quickly conduct performance analysis. In particular, a design of experiment (DOE) using an analytical model could help understand the relative importance of factors such as detection range, speed, and number of weapons carried. The understanding gained from relatively simple analytical models can help verify more complex simulations.

Unfortunately, the small progress made in this report falls well short of a comprehensive analytical modeling toolbox. In particular a many-on-many sequential engagement model is missing. The limited amount of work done on developing a many-on-many sequential engagement model reveals some of the pitfalls of analytical modeling. For example, unsatisfactory assumptions are often necessary to obtain a closed-form solution. Nevertheless, the special cases where closed-form solutions exist can be used to validate more general spreadsheet models.

8. Acronyms

AAM - air-to-air missile
AR - active radar
BVR - beyond visual range
DOE - design of experiments
LOAL - lock-on after launch
LOBL - lock-on before launch
LOS - line of sight
PKSS - probability of kill single shot
SAR - semi-active radar
TOF - time of flight

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