

2^kr Factorial Designs

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These slides are available on-line at:

<http://www.cse.wustl.edu/~jain/cse567-17/>



- ❑ Computation of Effects
- ❑ Estimation of Experimental Errors
- ❑ Allocation of Variation
- ❑ Confidence Intervals for Effects
- ❑ Confidence Intervals for Predicted Responses
- ❑ Visual Tests for Verifying the assumptions
- ❑ Multiplicative Models

2^kr Factorial Designs

- ❑ r replications of 2^k Experiments
 ⇒ 2^k r observations.
 ⇒ Allows estimation of experimental errors.

- ❑ Model:

$$y = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B + e$$

- ❑ e = Experimental error

Computation of Effects

Simply use means of r measurements

I	A	B	A B	y	Mean \bar{y}
1	-1	-1	1	(15, 18, 12)	15
1	1	-1	-1	(45, 48, 51)	48
1	-1	1	-1	(25, 28, 19)	24
1	1	1	1	(75, 75, 81)	77
164	86	38	20		total
41	21.5	9.5	5		total/4

- ❑ Effects: $q_0 = 41$, $q_A = 21.5$, $q_B = 9.5$, $q_{AB} = 5$.

Estimation of Experimental Errors

- Estimated Response:

$$\hat{y}_i = q_0 + q_A x_{Ai} + q_B x_{Bi} + q_{AB} x_{Ai} x_{Bi}$$

Experimental Error = Measured - Estimated

$$\begin{aligned} e_{ij} &= y_{ij} - \hat{y}_i \\ &= y_{ij} - q_0 - q_A x_{Ai} - q_B x_{Bi} - q_{AB} x_{Ai} x_{Bi} \\ \sum_{i,j} e_{ij} &= 0 \end{aligned}$$

$$\text{Sum of Squared Errors: } SSE = \sum_{i=1}^{2^2} \sum_{j=1}^r e_{ij}^2$$

Experimental Errors: Example

- Estimated Response:

$$\hat{y}_1 = q_0 - q_A - q_B + q_{AB} = 41 - 21.5 - 9.5 + 5 = 15$$

- Experimental errors:

$$e_{11} = y_{11} - \hat{y}_1 = 15 - 15 = 0$$

i	Effect				Estimated	Measured			Errors		
	I	A	B	A B	Response	y_{i1}	y_{i2}	y_{i3}	e_{i1}	e_{i2}	e_{i3}
	41	21.5	9.5	5	\hat{y}_i						
1	1	-1	-1	1	15	15	18	12	0	3	-3
2	1	1	-1	-1	48	45	48	51	-3	0	3
3	1	-1	1	-1	24	25	28	19	1	4	-5
4	1	1	1	1	77	75	75	81	-2	-2	4

Allocation of Variation

- Total variation or total sum of squares:

$$SST = \sum_{i,j} (y_{ij} - \bar{y}_{..})^2$$

$$y_{ij} = q_0 + q_A x_{Ai} + q_B x_{Bi} + q_{AB} x_{Ai} x_{Bi} + e_{ij}$$

$$\begin{aligned} \sum_{i,j} (y_{ij} - \bar{y}_{..})^2 &= 2^2 r q_A^2 + 2^2 r q_B^2 + 2^2 r q_{AB}^2 + \sum_{i,j} e_{ij}^2 \\ SST &= SSA + SSB + SSAB + SSE \end{aligned}$$

Example 18.3: Memory-Cache Study

$$\begin{aligned} SSY &= 15^2 + 18^2 + 12^2 + 45^2 + \dots + 75^2 + 75^2 + 81^2 \\ &= 27204 \end{aligned}$$

$$SS0 = 2^2 r q_0^2 = 12 \times 41^2 = 20172$$

$$SSA = 2^2 r q_A^2 = 12 \times (21.5)^2 = 5547$$

$$SSB = 2^2 r q_B^2 = 12 \times (9.5)^2 = 1083$$

$$SSAB = 2^2 r q_{AB}^2 = 12 \times 5^2 = 300$$

$$\begin{aligned} SSE &= 27204 - 2^2 \times 3(41^2 + 21.5^2 + 9.5^2 + 5^2) \\ &= 102 \end{aligned}$$

$$SST = SSY - SS0$$

$$= 27204 - 20172 = 7032$$

Example 18.3 (Cont)

$$\begin{aligned} \text{SSA} + \text{SSB} + \text{SSAB} + \text{SSE} \\ = 5547 + 1083 + 300 + 102 \\ = 7032 = \text{SST} \end{aligned}$$

Factor A explains 5547/7032 or 78.88%

Factor B explains 15.40%

Interaction AB explains 4.27%

1.45% is unexplained and is attributed to errors.

Confidence Intervals For Effects

- Effects are random variables.
- Errors $\sim N(0, \sigma_e) \Rightarrow y \sim N(\bar{y}, \sigma_e)$

$$q_0 = \frac{1}{2^2 r} \sum_{i,j} y_{ij}$$

- q_0 = Linear combination of normal variates
 $\Rightarrow q_0$ is normal with variance $\sigma_e^2/(2^2 r)$

Variance of errors:

$$s_e^2 = \frac{1}{2^2(r-1)} \sum_{ij} e_{ij}^2 = \frac{\text{SSE}}{2^2(r-1)} \triangleq \text{MSE}$$

- Denominator = $2^2(r-1)$ = # of independent terms in SSE
 \Rightarrow SSE has $2^2(r-1)$ degrees of freedom.
 Estimated variance of q_0 : $s_{q_0}^2 = s_e^2/(2^2 r)$

Conf. Intervals For Effects (Cont)

- Similarly,

$$s_{q_A} = s_{q_B} = s_{q_{AB}} = \frac{s_e}{\sqrt{2^2 r}}$$

- Confidence intervals (CI) for the effects:

$$q_i \mp t_{[1-\alpha/2; 2^2(r-1)]} s_{q_i}$$

- CI does not include a zero \Rightarrow significant

Example 18.4

- For Memory-cache study: Standard deviation of errors:

$$s_e = \sqrt{\frac{\text{SSE}}{2^2(r-1)}} = \sqrt{\frac{102}{8}} = \sqrt{12.75} = 3.57$$

- Standard deviation of effects:

$$s_{q_i} = s_e / \sqrt{2^2 r} = 3.57 / \sqrt{12} = 1.03$$

- For 90% Confidence: $t_{[0.95, 8]} = 1.86$

- Confidence intervals: $q_i \mp (1.86)(1.03) = q_i \mp 1.92$

$$q_0 = (39.08, 42.91)$$

$$q_A = (19.58, 23.41)$$

$$q_B = (7.58, 11.41)$$

$$q_{AB} = (3.08, 6.91)$$

- No zero crossing \Rightarrow All effects are significant.

Confidence Intervals for Contrasts

- Contrast: Linear combination with \sum coefficients = 0

$$\sum h_i q_i \text{ with } \sum h_i = 0$$

For example, $q_A - q_B$ or $q_A + q_B - 2q_{AB}$

- Mean of $\sum h_i q_i = \sum h_i E[q_i]$

- Variance of $\sum h_i q_i$ $s_{\sum h_i q_i}^2 = \frac{s_e^2 \sum h_i^2}{2^2 r}$

- For 100(1- α)% confidence interval, use $t_{[1-\alpha/2; 2^2(r-1)]}$.

Example 18.5

Memory-cache study

$$u = q_A + q_B - 2q_{AB}$$

Coefficients = 0, 1, 1, and -2 \Rightarrow Contrast

$$\text{Mean } \bar{u} = 21.5 + 9.5 - 2 \times 5 = 21$$

$$\text{Variance } s_u^2 = \frac{s_e^2 \times 6}{2^2 \times 3} = 6.375$$

$$\text{Standard deviation } s_u = \sqrt{6.375} = 2.52$$

$$t_{[0.95; 8]} = 1.86$$

90% Confidence interval for u:

$$\bar{u} \mp t s_u = 21 \mp 1.86 \times 2.52 = (16.31, 25.69)$$

Conf. Interval For Predictions

- Mean response \hat{y} :

$$\hat{y} = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B$$

- The standard deviation of the mean of m responses:

$$s_{\hat{y}_m} = s_e \left(\frac{1}{n_{\text{eff}}} + \frac{1}{m} \right)^{1/2}$$

$$n_{\text{eff}} = \text{Effective deg of freedom}$$

$$\begin{aligned} &= \frac{\text{Total number of runs}}{1 + \text{Sum of DFs of params used in } \hat{y}} \\ &= \frac{2^2 r}{5} \end{aligned}$$

Conf. Interval for Predictions (Cont)

100(1- α)% confidence interval:

$$\hat{y} \mp t_{[1-\alpha/2; 2^2(r-1)]} s_{\hat{y}_m}$$

- A single run (m=1): $s_{\hat{y}_1} = s_e \left(\frac{5}{2^2 r} + 1 \right)^{1/2}$

- Population mean (m= ∞): $s_{\hat{y}} = s_e \left(\frac{5}{2^2 r} \right)^{1/2}$

Example 18.6: Memory-cache Study

- For $x_A = -1$ and $x_B = -1$:
- A single confirmation experiment:

$$\begin{aligned}\hat{y}_1 &= q_0 - q_A - q_B + q_{AB} \\ &= 41 - 21.5 - 9.5 + 5 = 15\end{aligned}$$

- Standard deviation of the prediction:

$$s_{\hat{y}_1} = s_e \left(\frac{5}{2^2 r} + 1 \right)^{1/2} = 3.57 \sqrt{\frac{5}{12} + 1} = 4.25$$

- Using $t_{[0.95;8]} = 1.86$, the 90% confidence interval is:

$$15 \mp 1.86 \times 4.25 = (7.09, 22.91)$$

Example 18.6 (Cont)

- Mean response for 5 experiments in future:

$$\begin{aligned}s_{\hat{y}_1} &= s_e \left(\frac{5}{2^2 r} + \frac{1}{m} \right)^{1/2} \\ &= 3.57 \sqrt{\frac{5}{12} + \frac{1}{5}} = 2.80\end{aligned}$$

- The 90% confidence interval is:

$$15 \mp 1.86 \times 2.80 = (9.79, 20.21)$$

Example 18.6 (Cont)

- Mean response for a large number of experiments in future:

$$s_{\hat{y}_1} = s_e \left(\frac{5}{2^2 r} \right)^{1/2} = 3.57 \sqrt{\frac{5}{12}} = 2.30$$

- The 90% confidence interval is:

$$15 \mp 1.86 \times 2.30 = (10.72, 19.28)$$

- Current mean response: Not for future. Use contrasts formula.

$$s_{\hat{y}_1} = \sqrt{\frac{s_e^2 \sum h_i^2}{2^2 r}} = \sqrt{\frac{12.75 \times 4}{12}} = 2.06$$

- 90% confidence interval:

$$15 \mp 1.86 \times 2.06 = (11.17, 18.83)$$

Homework 18A

Updated Exercise 18.1: The following table lists measured CPU times for two processors on two workloads. Each experiment was repeated three times. Determine the effects.

Table 18.12 $2^2 \times 3$ Experimental Design Exercise

Workload	Processor	
	A	B
I	(41.16, 39.02, 42.56)	(65.17, 69.25, 64.23)
J	(53.50, 55.50, 50.50)	(50.08, 48.98, 47.10)

Assumptions

1. Errors are statistically independent.
2. Errors are additive.
3. Errors are normally distributed.
4. Errors have a constant standard deviation σ_e .
5. Effects of factors are additive
 \Rightarrow observations are independent and normally distributed with constant variance.

Visual Tests

1. Independent Errors:

- ❑ Scatter plot of residuals versus the predicted response \hat{y}_i
- ❑ Magnitude of residuals < Magnitude of responses/10
 \Rightarrow Ignore trends
- ❑ Plot the residuals as a function of the experiment number
- ❑ Trend up or down \Rightarrow other factors or side effects

2. Normally distributed errors:

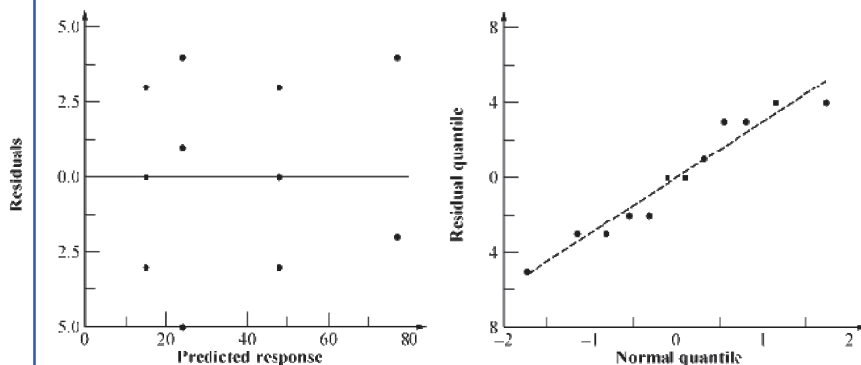
Normal quantile-quantile plot of errors

3. Constant Standard Deviation of Errors:

Scatter plot of y for various levels of the factor

Spread at one level significantly different than that at other
 \Rightarrow Need transformation

Example 18.7: Memory-cache



Multiplicative Models

❑ Additive model:

$$y_{ij} = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B + e_{ij}$$

❑ Not valid if effects do not add.

E.g., execution time of workloads.

i th processor speed = v_i instructions/second.

j th workload Size = w_j instructions

❑ The two effects multiply. Logarithm \Rightarrow additive model:

Execution Time $y_{ij} = w_j / v_i$

$$\log(y_{ij}) = \log(w_j) - \log(v_i)$$

❑ Correct Model:

$$y'_{ij} = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B + e_{ij}$$

Where, $y'_{ij} = \log(y_{ij})$

Multiplicative Model (Cont)

- Taking an antilog of effects:

$$u_A = 10^{q_A}, u_B = 10^{q_B}, \text{ and } u_{AB} = 10^{q_{AB}}$$

- u_A = ratio of MIPS rating of the two processors
- u_B = ratio of the size of the two workloads.
- Antilog of additive mean $q_0 \Rightarrow$ geometric mean

$$\hat{y} = 10^{q_0} = (y_1 y_2 \cdots y_n)^{1/n} \quad n = 2^r$$

Example 18.8: Execution Times

Analysis Using an Additive Model

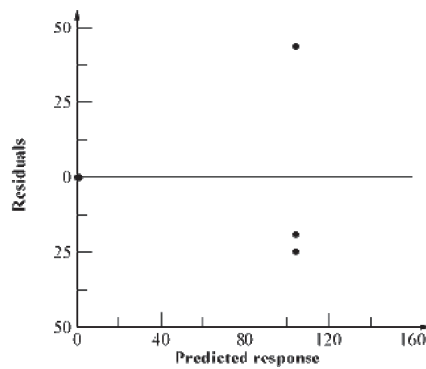
I	A	B	AB	y	Mean \bar{y}
1	-1	-1	1	(85.10, 79.50, 147.90)	104.170
1	1	-1	-1	(0.891, 1.047, 1.072)	1.003
1	-1	1	-1	(0.955, 0.933, 1.122)	1.003
1	1	1	1	(0.0148, 0.0126, 0.0118)	0.013
106.19	-104.15	-104.15	102.17	total	
26.55	-26.04	-26.04	25.54	total/4	

Additive model is not valid because:

- Physical consideration \Rightarrow effects of workload and processors do not add. They multiply.
- Large range for y. $y_{\max}/y_{\min} = 147.90/0.0118$ or 12,534
 \Rightarrow log transformation
- Taking an arithmetic mean of 104.17 and 0.013 is inappropriate.

Example 18.8 (Cont)

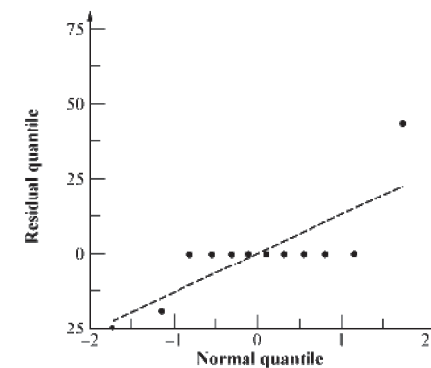
- The residuals are not small as compared to the response.



- The spread of residuals is large at larger value of the response.
 \Rightarrow log transformation

Example 18.8 (Cont)

- Residual distribution has a longer tail than normal



Analysis Using Multiplicative Model

Data After Log Transformation						
I	A	B	AB	y	Mean \bar{y}	
1	-1	-1	1	(1.93, 1.90, 2.17)	2.00	
1	1	-1	-1	(-0.05, 0.02, 0.03)	0.00	
1	-1	1	-1	(-0.02, -0.03, 0.05)	0.00	
1	1	1	1	(-1.83, -1.90, -1.93)	-1.89	
0.11	-3.89	-3.89	0.11			total
0.03	-0.97	-0.97	0.03			total/4

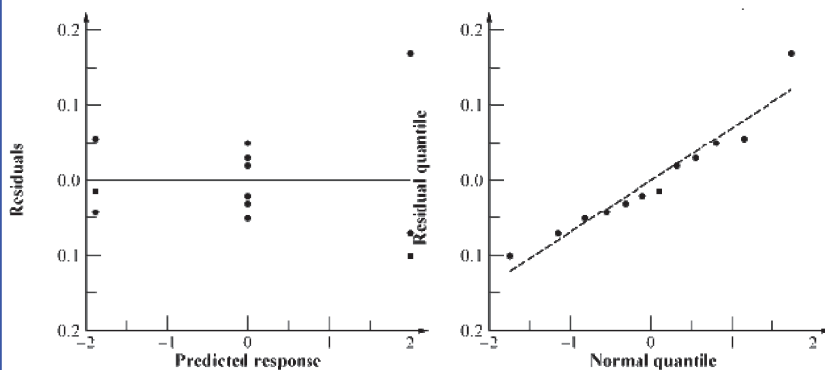
Variation Explained by the Two Models

Factor	Additive Model			Multiplicative Model		
	Effect	% Var.	Conf. Interval	Effect	% Var.	Conf. Interval
I	26.55		(16.35, 36.74)	0.03		(-0.02, 0.07)†
A	-26.04	30.1%	(-36.23, -15.84)	-0.97	49.9%	(-1.02, -0.93)
B	-26.04	30.1%	(-36.23, -15.84)	-0.97	49.9%	(-1.02, -0.93)
AB	25.54	29.0%	(15.35, 35.74)	0.03	0.0%	(-0.02, 0.07)†
e		10.8%			0.2%	

† ⇒ Not Significant

- With multiplicative model:
 - Interaction is almost zero.
 - Unexplained variation is only 0.2%

Visual Tests



- **Conclusion:** Multiplicative model is better than the additive model.

Interpretation of Results

$$\log(y) = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B + e$$

$$\Rightarrow y = 10^{q_0} 10^{q_A x_A} 10^{q_B x_B} 10^{q_{AB} x_A x_B} 10^e$$

$$= 10^{0.03} 10^{-0.97 x_A} 10^{-0.97 x_B} 10^{0.03 x_A x_B} 10^e$$

$$= 1.07 \times 0.107^{x_A} \times 0.107^{x_B} \times 1.07^{x_A x_B} 10^e$$

- The time for an average processor on an average benchmark is 1.07.
- The time on processor A_1 is 9.35 times (0.107^{-1}) that on an average processor. The time on A_2 is $1/9.35$ (0.107^1) of that on an average processor.
- MIPS rate for A_2 is 87.4 times that of A_1 .
- Benchmark B_1 executes 87.4 times more instructions than B_2 .
- The interaction is negligible.

⇒ Results apply to all benchmarks and processors.

Transformation Considerations

- y_{\max}/y_{\min} small \Rightarrow Multiplicative model results similar to additive model.
- Many other transformations possible.
- Box-Cox family of transformations:

$$w = \begin{cases} \frac{y^a - 1}{ag^{a-1}}, & a \neq 0 \\ (\ln y)g, & a = 0 \end{cases}$$
- Where g is the geometric mean of the responses:

$$g = (y_1 y_2 \cdots y_n)^{1/n}$$
- w has the same units as y .
- a can have any real value, positive, negative, or zero.
- Plot SSE as a function of $a \Rightarrow$ optimal a
- Knowledge about the system behavior should always take precedence over statistical considerations.

General 2^k r Factorial Design

- Model:

$$y_{ij} = q_0 + q_A x_{Ai} + q_B x_{Bi} + q_{AB} x_{Ai} x_{Bi} + \cdots + e_{ij}$$

- Parameter estimation:

$$q_j = \frac{1}{2^k} \sum_{i=1}^{2^k} S_{ij} \bar{y}_i$$

S_{ij} = (i,j) th entry in the sign table.

- Sum of squares:

$$SSY = \sum_{i=1}^{2^k} \sum_{j=1}^r y_{ij}^2$$

$$SS0 = 2^k r q_0^2$$

$$SST = SSY - SS0$$

$$SSj = 2^k r q_j^2 \quad j = 1, 2, \dots, 2^k - 1$$

$$SSE = SST - \sum_{j=1}^{2^k-1} SSj$$

$$\begin{aligned} SST &= SSY - SS0 = \sum_{j=1}^{2^k} SSj + SSE \\ 2^k r - 1 &= 2^k r - 1 = \sum_{j=1}^{2^k} 1 + 2^k (r - 1) \end{aligned}$$

General 2^k r Factorial Design (Cont)

- Percentage of y 's variation explained by j th effect =

$$(SSj/SST) \times 100\%$$

- Standard deviation of errors:

$$s_e = \sqrt{\frac{SSE}{2^k(r-1)}}$$

- Standard deviation of effects:

$$s_{q_0} = s_{q_A} = s_{q_B} = s_{q_{AB}} = s_e / \sqrt{2^k r}$$

- Variance of contrast $\sum h_i q_i$, where $\sum h_i = 0$ is:

$$s_{\sum h_i q_i}^2 = (s_e^2 \sum h_i^2) / 2^k r$$

General 2^k r Factorial Design (Cont)

- Standard deviation of the mean of m future responses:

$$s_{\hat{y}_p} = s_e \left(\frac{1 + 2^k}{2^k r} + \frac{1}{m} \right)^{1/2}$$

- Confidence intervals are calculated using $t_{[1-\alpha/2; 2^k(r-1)]}$.

- Modeling assumptions:

- Errors are IID normal variates with zero mean.
- Errors have the same variance for all values of the predictors.
- Effects and errors are additive.

Visual Tests for 2^kr Designs

- ❑ The scatter plot of errors versus predicted responses should not have any trend.
- ❑ The normal quantile-quantile plot of errors should be linear.
- ❑ Spread of y values in all experiments should be comparable.

Example 18.9: A 2³ Design

I	A	B	C	AB	AC	BC	ABC	y	Mean \bar{y}
1	-1	-1	-1	1	1	1	-1	(14, 16, 12)	14
1	1	-1	-1	-1	-1	1	1	(22, 18, 20)	20
1	-1	1	-1	-1	1	-1	1	(11, 15, 19)	15
1	1	1	-1	1	-1	-1	-1	(34, 30, 35)	33
1	-1	-1	1	1	-1	-1	1	(46, 42, 44)	44
1	1	-1	1	-1	1	-1	-1	(58, 62, 60)	60
1	-1	1	1	-1	-1	1	-1	(50, 55, 54)	53
1	1	1	1	1	1	1	1	(86, 80, 74)	80
319	67	43	155	23	19	15	-1	total	
39.87	8.375	5.375	19.37	2.875	2.375	1.875	-0.125	total/8	

Example 18.9 (Cont)

- ❑ Sum of Squares:

Component	Sum of Squares	Percent Variation
y	4.9E4	
\bar{y}	3.8E4	
y- \bar{y}	1.1E4	100.00%
A	1683.0	14.06%
B	693.3	5.79%
C	9009.0	75.27%
AB	198.3	1.66%
AC	135.4	1.13%
BC	84.4	0.70%
ABC	0.4	0.00%
Errors	164.0	1.37%

Example 18.9 (Cont)

- ❑ The errors have 2³(3-1) or 16 degrees of freedom. Standard deviation of errors:

$$s_e = \sqrt{\frac{\text{SSE}}{2^k(r-1)}} = \sqrt{\frac{164}{16}} = 3.20$$

- ❑ Standard deviation of effects:

$$s_{q_i} = s_e / \sqrt{(2^3 3)} = 3.20 / \sqrt{24} = 0.654$$

Example 18.9 (Cont)

□ % Variation:

Component	Sum of Squares	Percent Variation
y	4.9E4	
\bar{y}	3.8E4	
$y-\bar{y}$	1.1E4	100.00%
A	1683.0	14.06%
B	693.3	5.79%
C	9009.0	75.27%
AB	198.3	1.66%
AC	135.4	1.13%
BC	84.4	0.70%
ABC	0.4	0.00%
Errors	164.0	1.37%

Example 18.9 (Cont)

□ $t_{[0.90, 16]}=1.337$

□ 80% confidence intervals for parameters: $q_i \mp (1.337)(0.654)$
 $= q_i \mp 0.874$

$q_0 = (39.00, 40.74)$

$q_A = (7.50, 9.25)$

$q_B = (4.50, 6.25)$

$q_C = (18.50, 20.24)$

$q_{AB} = (2.00, 3.75)$

$q_{AC} = (1.50, 3.25)$

$q_{BC} = (1.00, 2.75)$

$q_{ABC} = (-1.00, 0.75)$

□ All effects except q_{ABC} are significant.

Example 18.9 (Cont)

□ For a single confirmation experiment ($m = 1$)

With $A = B = C = -1$:

$$\hat{y} = 14$$

$$s_{\hat{y}} = s_e \left(\frac{9}{2^k r} + \frac{1}{m} \right)^{1/2}$$

$$= 3.2 \left(\frac{9}{24} + 1 \right)^{1/2}$$

$$= 3.75$$

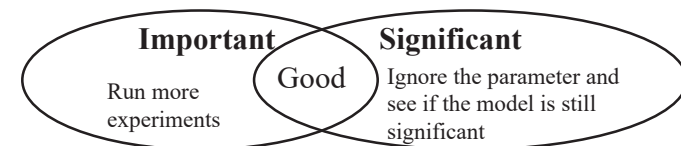
□ 80% confidence interval:

$$14 \mp 1.337 \times 3.75 = 14 \mp 5.02 = (8.98, 19.02)$$

Importance vs. Significance

□ Important = Parameters or models that explain a high percent of variation

□ Significant = Parameters or models whose confidence interval does not include zero



Case Study 18.1: Garbage collection

Factors and Levels

Variable	Factor	-1	1
A	Workload	Single Task	Several parallel tasks
B	Compiler	Simple	Deallocating
C	Limbo List	Enabled	Disabled
D	Chunk Size	4K bytes	16K bytes

Case Study 18.1 (Cont)

I	A	B	C	D	y	Mean \bar{y}				
1	-1	-1	-1	-1	(97, 97, 97)	97.00				
1	1	-1	-1	-1	(31, 31, 32)	31.33				
1	-1	1	-1	-1	(97, 97, 97)	97.00				
1	1	1	-1	-1	(31, 32, 31)	31.33				
1	-1	-1	1	-1	(97, 97, 97)	97.00				
1	1	-1	1	-1	(32, 32, 31)	31.67				
1	-1	1	1	-1	(97, 97, 97)	97.00				
1	1	1	1	-1	(32, 32, 32)	32.00				
1	-1	-1	-1	1	(407, 407, 407)	407.00				
1	1	-1	-1	1	(135, 136, 135)	135.33				
1	-1	1	-1	1	(409, 409, 409)	409.00				
1	1	1	-1	1	(135, 135, 136)	135.33				
1	-1	-1	1	1	(407, 407, 407)	407.00				
1	1	-1	1	1	(139, 140, 139)	139.33				
1	-1	1	1	1	(409, 409, 409)	409.00				
1	1	1	1	1	(139, 139, 140)	139.33				
					2695.67	-1344.33	4.33	9.00	1667.00	total
					168.48	-84.02	0.27	0.56	104.19	total/16

Case Study 18.1 (Cont)

Factor	Effect	% Variation	Conf. Interval
I	168.48		(168.386, 168.573)
A	-84.02	34.4%	(-84.114, -83.927)
B	0.27	0.0%	(0.177, 0.364)
C	0.56	0.0%	(0.469, 0.656)
D	104.19	52.8%	(104.094, 104.281)
AB	-0.23	0.0%	(-0.323, -0.136)
AC	0.56	0.0%	(0.469, 0.656)
AD	-51.31	12.8%	(-51.406, -51.219)
BC	0.02	0.0%	(-0.073, 0.114)†
BD	0.23	0.0%	(0.136, 0.323)
CD	0.44	0.0%	(0.344, 0.531)
ABC	0.02	0.0%	(-0.073, 0.114)†
ABD	-0.27	0.0%	(-0.364, -0.177)
ACD	0.44	0.0%	(0.344, 0.531)
BCD	-0.02	0.0%	(-0.114, 0.073)†
ABCD	-0.02	0.0%	(-0.114, 0.073)†

† ⇒ Not Significant

Case Study 18.1: Conclusions

- ❑ Most of the variation is explained by factors A (Workload), D (Chunk size), and the interaction A D between the two.
- ❑ The variation due to experimental error is small
⇒ Several effects that explain less than 0.05% of variation (listed as 0.0%) are statistically significant.
- ❑ Only effects A, D, and AD are both practically significant and statistically significant.

Summary



- ❑ Replications allow estimation of measurement errors
 - ⇒ Confidence Intervals of parameters
 - ⇒ Confidence Intervals of predicted responses
- ❑ Allocation of variation is proportional to square of effects
- ❑ Multiplicative models are appropriate if the factors multiply
- ❑ Visual tests for independence normal errors

Homework 18B

Updated Exercise 18.1: For the data of Homework 18A, determine percentage of variation explained, find confidence intervals of the effects, and conduct visual tests.

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