Analysis of Simulation Results

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Audio/Video recordings of this lecture are available at:

http://www.cse.wustl.edu/~jain/cse567-17/

- Analysis of Simulation Results
- Model Verification Techniques
- Model Validation Techniques
- Transient Removal
- Terminating Simulations
- Stopping Criteria: Variance Estimation
- Variance Reduction

Model Verification vs. Validation

- Verification ⇒ Debugging
- Validation ⇒ Model = Real world

- Four Possibilities:
  1. Unverified, Invalid
  2. Unverified, Valid
  3. Verified, Invalid
  4. Verified, Valid

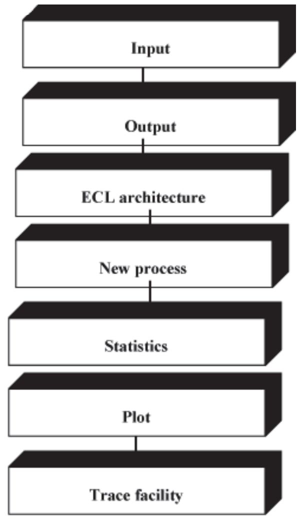
Model Verification Techniques

1. Top Down Modular Design
2. Anti-bugging
3. Structured Walk-Through
4. Deterministic Models
5. Run Simplified Cases
6. Trace
7. On-Line Graphic Displays
8. Continuity Test
9. Degeneracy Tests
10. Consistency Tests
11. Seed Independence

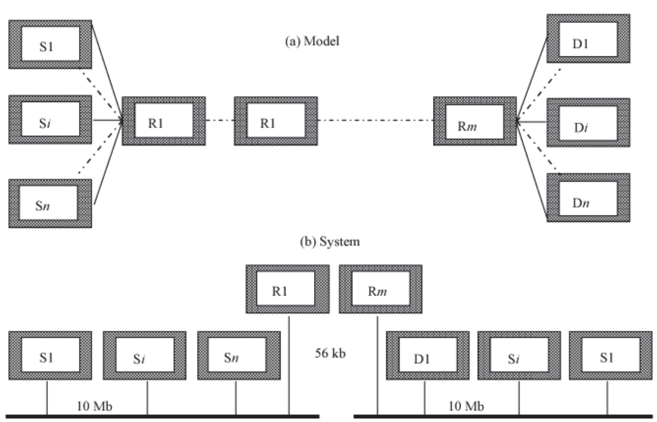
Top Down Modular Design

- Divide and Conquer
- Modules = Subroutines, Subprograms, Procedures
  - Modules have well defined interfaces
  - Can be independently developed, debugged, and maintained
- Top-down design
  ⇒ Hierarchical structure
  ⇒ Modules and sub-modules

Top Down Modular Design (Cont)



Top Down Modular Design (Cont)



Verification Techniques

- **Anti-bugging**: Include self-checks:
  å Probabilities = 1
  Jobs left = Generated - Serviced
- **Structured Walk-Through**:
  - Explain the code another person or group
  - Works even if the person is sleeping
- **Deterministic Models**: Use constant values
- **Run Simplified Cases**:
  - Only one packet
  - Only one source
  - Only one intermediate node

Trace

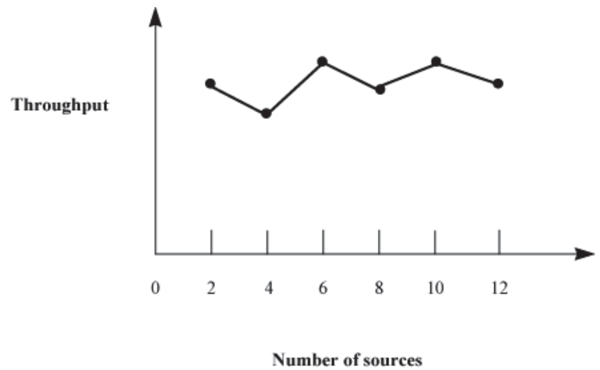
- Trace = Time-ordered list of events and variables
- Several levels of detail:
  - Events trace
  - Procedure trace
  - Variables trace
- User selects the detail
  - Include on and off
- See Fig 25.3 in the Text Book on page 418 for a sample trace

On-Line Graphic Displays

- Make simulation interesting
- Help selling the results
- More comprehensive than trace

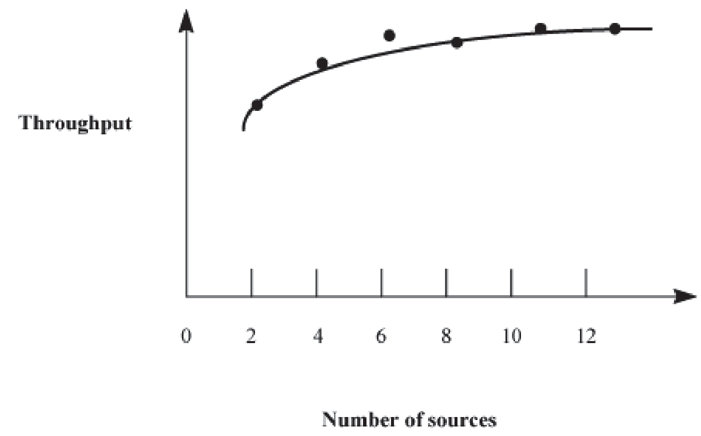
Continuity Test

- Run for different values of input parameters
- Slight change in input ⇒ slight change in output
- Before:



Continuity Test (Cont)

- After:



More Verification Techniques

- **Degeneracy Tests**: Try extreme configuration and workloads
    - One CPU, Zero disk
- **Consistency Tests**:
  - Similar result for inputs that have same effect
    - Four users at 100 Mbps vs. Two at 200 Mbps
  - Build a test library of continuity, degeneracy and consistency tests
- **Seed Independence**: Similar results for different seeds

Model Validation Techniques

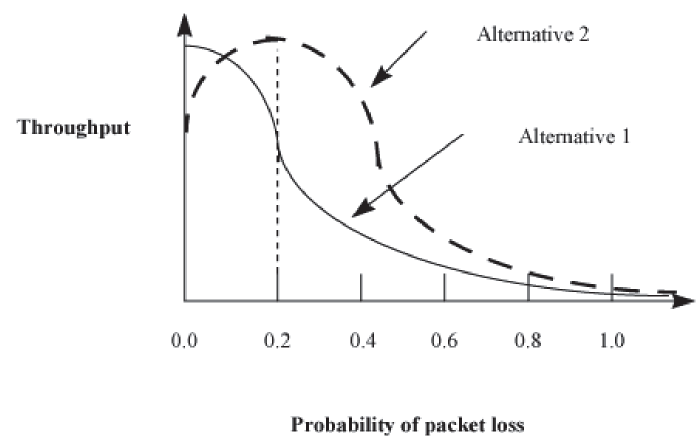
- Validation techniques for one problem may not apply to another problem.
- Aspects to Validate:
  1. Assumptions
  2. Input parameter values and distributions
  3. Output values and conclusions
- Techniques:
  1. Expert intuition
  2. Real system measurements
  3. Theoretical results

$\Rightarrow$ $3 \times 3 = 9$ validation tests

Expert Intuition

- Most practical and common way
- Experts = Involved in design, architecture, implementation, analysis, marketing, or maintenance of the system
- Selection = fn of Life cycle stage
- Present assumption, input, output
- Better to validate one at a time
- See if the experts can distinguish simulation vs. measurement

Expert Intuition (Cont)



Real System Measurements

- Compare assumptions, input, output with the real world
- Often infeasible or expensive
- Even one or two measurements add to the validity

Theoretical Results

- Analysis = Simulation
- Used to validate analysis also
- Both may be invalid
- Use theory in conjunction with experts' intuition
  - E.g., Use theory for a large configuration
  - Can show that the model is not invalid

Exercise 25.1

Imagine that you have been called as an expert to review a simulation study. Which of the following simulation results would you consider non-intuitive and would want it carefully validated:

1. The throughput of a system increases as its load increases.
2. The throughput of a system decreases as its load increases.
3. The response time increases as the load increases.
4. The response time of a system decreases as its load increases.
5. The loss rate of a system decreases as the load increases.

This is not a homework. Try but do not submit. Check answer in the book.

Transient Removal

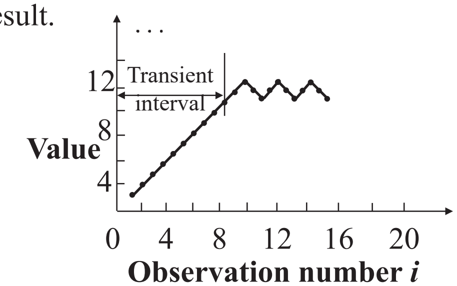
- Generally steady state performance is interesting
- Remove the initial part
- No exact definition ⇒ Heuristics:
  1. Long Runs
  2. Proper Initialization
  3. Truncation
  4. Initial Data Deletion
  5. Moving Average of Independent Replications
  6. Batch Means

Transient Removal Techniques

- **Long Runs**:
  - Wastes resources
  - Difficult to insure that it is long enough
- **Proper Initialization**:
  - Start in a state close to expected steady state
    $\Rightarrow$ Reduces the length and effect of transient state

Truncation

- Assumes variability is lower during steady state
- Plot max-min of *n-l* observation for *l* =1, 2, ……….
- When *(l+1)*th observation is neither the minimum nor maximum $\Rightarrow$ transient state ended
- At l = 9, Range = (9, 11), next observation = 10
- Sometimes incorrect result.



Terminating Simulations

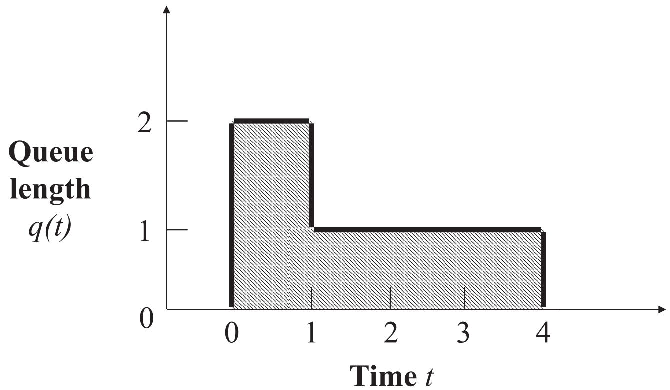
- Transient performance is of interest
  E.g., Network traffic
- System shuts down $\Rightarrow$ Do not need transient removal.
- Final conditions:
  - May need to exclude the final portion from results
  - Techniques similar to transient removal

Treatment of Leftover Entities

- Mean service time $= \dfrac{\text{Total service time}}{\text{Number of jobs that completed service}}$
- Mean waiting time $= \dfrac{\text{Sum of waiting time}}{\text{Number of jobs that received service}}$
- Mean Queue Length $\neq \dfrac{\sum_{j=1}^{n} \text{Queue length at event j}}{\text{Number of events } n}$

$$= \frac{1}{T}\int_0^T \text{Queue\_length}(t)dt$$

Example 25.3: Treatment of Leftover Entities



- Three events: Arrival at t=0, departures at t=1 and t=4
- Q = 2, 1, 0 at these events. Avg Q = (2+1+0)/3 = 1
- Avg Q = Area/4 = 5/4

Stopping Criteria: Variance Estimation

- Run until confidence interval is narrow enough

$$\bar{x} \pm z_{1-\alpha/2}\sqrt{\text{Var}(\bar{x})}$$

- For Independent observations:

$$\text{Var}(\bar{x}) = \frac{\text{Var}(x)}{n}$$

- Independence not applicable to most simulations.
- Large waiting time for ith job
  ⇒ Large waiting time for (i+1)th job
- For correlated observations:

$$\text{Actual variance} \gg \frac{\text{Var}(x)}{n}$$

Variance Estimation Methods

1. Independent Replications
2. Batch Means
3. Method of Regeneration

Independent Replications

- Assumes that means of independent replications are independent
- Conduct m replications of size $n+n_0$ each
  1. Compute a mean for each replication:

$$\bar{x}_i = \frac{1}{n}\sum_{j=n_0+1}^{n_0+n} x_{ij} \quad i = 1, 2, \ldots, m$$

  2. Compute an overall mean for all replications:

$$\bar{\bar{x}} = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_i$$

Independent Replications (Cont)

3. Calculate the variance of replicate means:

$$\text{Var}(\bar{x}) = \frac{1}{m-1}\sum_{i=1}^{m}(\bar{x}_i - \bar{\bar{x}})^2$$

4. Confidence interval for the mean response is:

$$\left[\bar{\bar{x}} \mp z_{1-\alpha/2}\sqrt{\text{Var}(\bar{x})/m}\right]$$

- Keep replications large to avoid waste
- Ten replications generally sufficient

Batch Means

- Also called method of sub-samples
- Run a long simulation run
- Discard initial transient interval, and Divide the remaining observations run into several batches or sub-samples.
  1. Compute means for each batch:

$$\bar{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij} \quad i = 1, 2, \ldots, m$$

  2. Compute an overall mean:

$$\bar{\bar{x}} = \frac{1}{m}\sum_{i=1}^{m}\bar{x}_i$$

Batch Means (Cont)

3. Calculate the variance of batch means:

$$\text{Var}(\bar{x}) = \frac{1}{m-1}\sum_{i=1}^{m}(\bar{x}_i - \bar{\bar{x}})^2$$

4. Confidence interval for the mean response is:

$$\left[\bar{\bar{x}} \mp z_{1-\alpha/2}\sqrt{\text{Var}(\bar{x})/m}\right]$$

- Less waste than independent replications
- Keep batches long to avoid correlation
- Check: Compute the auto-covariance of successive batch means:

$$\text{Cov}(\bar{x}_i, \bar{x}_{i+1}) = \frac{1}{m-2}\sum_{i=1}^{m-1}(\bar{x}_i - \bar{\bar{x}})(\bar{x}_{i+1} - \bar{\bar{x}})$$

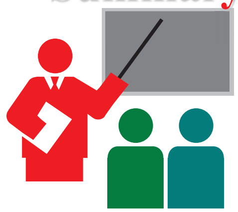
- Double n until autocovariance is small.

Case Study 25.1: Interconnection Networks

- Indirect binary n-cube networks: Used for processor-memory interconnection
- Two stage network with full fan out.
- At 64, autocovariance < 1% of sample variance

Batch Size	Autocovariance	Variance
1	-0.18792	1.79989
2	0.02643	0.81173
4	0.11024	0.42003
8	0.08979	0.26437
16	0.04001	0.17650
32	0.01108	0.10833
64	0.00010	0.06066
128	-0.00378	0.02992
256	0.00027	0.01133
512	0.00069	0.00503
1024	0.00078	0.00202

Summary



1. Verification = Debugging
   ⇒ Software development techniques
2. Validation ⇒ Simulation = Real ⇒ Experts involvement
3. Transient Removal: Initial data deletion, batch means
4. Terminating Simulations = Transients are of interest
5. Stopping Criteria: Independent replications, batch means, method of regeneration
6. Variance reduction is not for novice

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CSE567M: Computer Systems Analysis (Spring 2013),
https://www.youtube.com/playlist?list=PLjGG94etKypJEKjNAa1n_1X0bWWNyZcof

CSE473S: Introduction to Computer Networks (Fall 2011),
https://www.youtube.com/playlist?list=PLjGG94etKypJWOSPMh8Azcgy5e_10TiDw

Wireless and Mobile Networking (Spring 2016),
https://www.youtube.com/playlist?list=PLjGG94etKypKeb0nzyN9tSs_HCd5c4wXF

CSE571S: Network Security (Fall 2011),
https://www.youtube.com/playlist?list=PLjGG94etKypKvzfVtutHcPFJXumyyg93u

Video Podcasts of Prof. Raj Jain's Lectures,
https://www.youtube.com/channel/UCN4-5wzNP9-ruOzQMs-8NUw