



Queuing Networks

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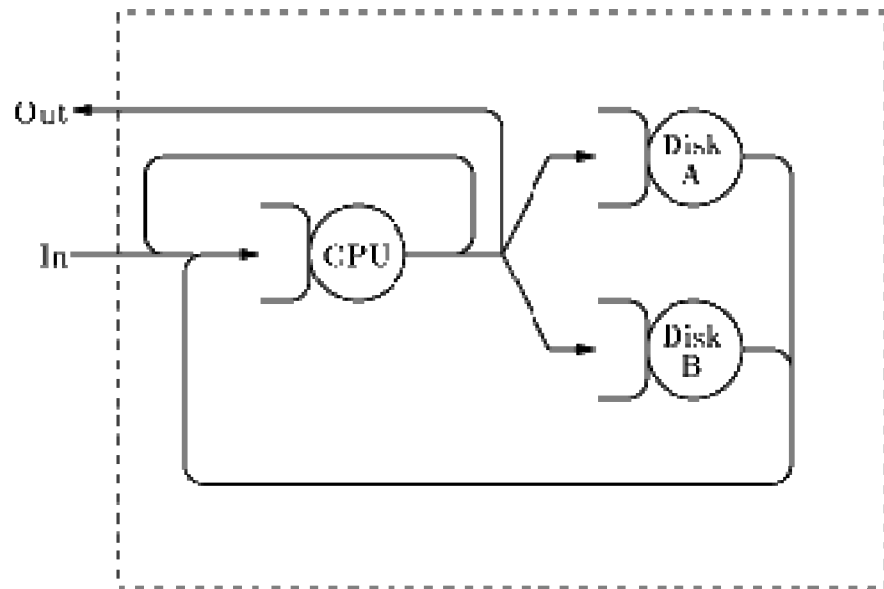
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1. Open and Closed Queueing Networks
2. Product Form Networks
3. Queueing Network Models of Computer Systems

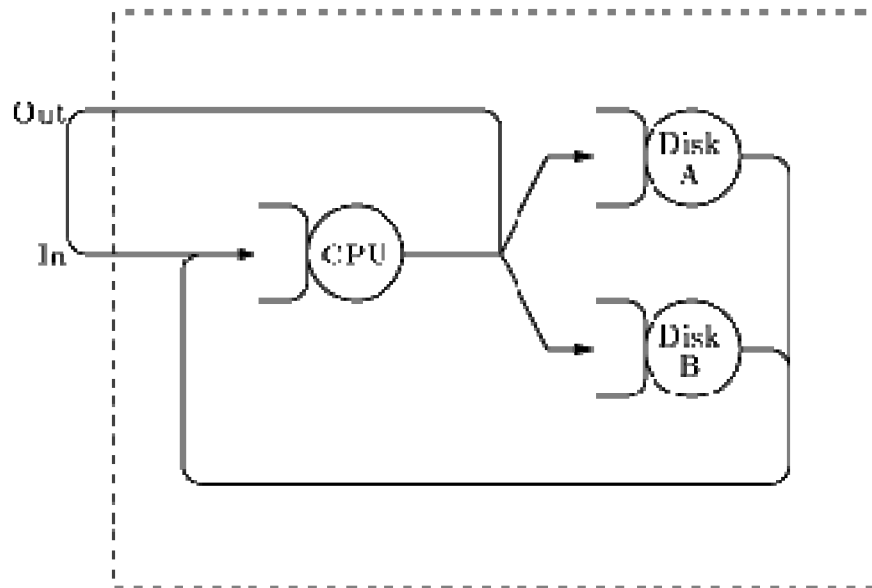
Open Queueing Networks

- ❑ **Queueing Network:** model in which jobs departing from one queue arrive at another queue (or possibly the same queue)
- ❑ **Open queueing network:** external arrivals and departures
 - Number of jobs in the system varies with time.
 - Throughput = arrival rate
 - Goal: To characterize the distribution of number of jobs in the system.



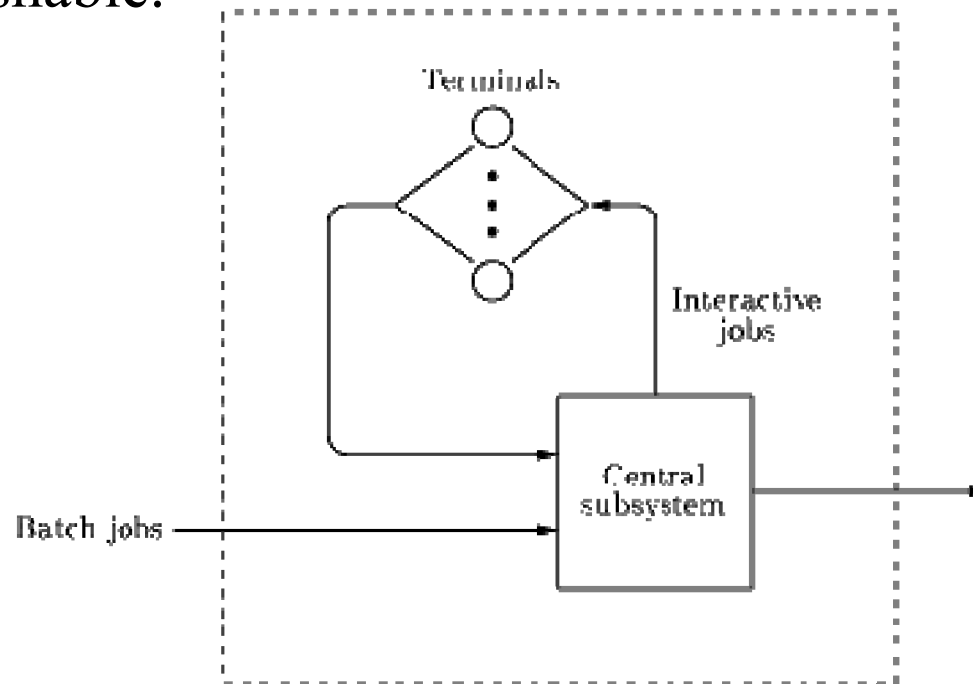
Closed Queueing Networks

- ❑ Closed queueing network: No external arrivals or departures
 - Total number of jobs in the system is constant
 - 'OUT' is connected back to 'IN.'
 - Throughput = flow of jobs in the OUT-to-IN link
 - Number of jobs is given, determine the throughput

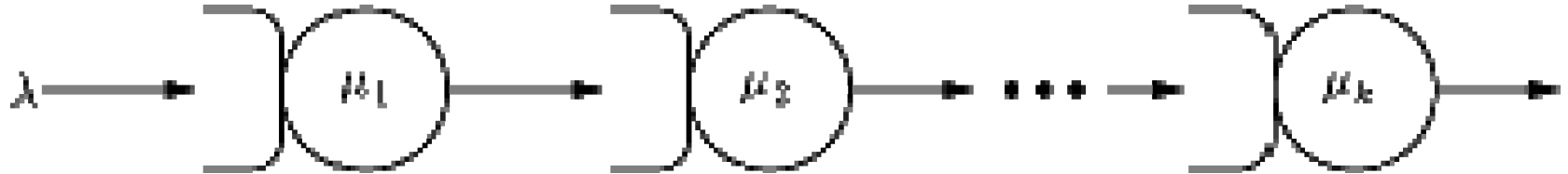


Mixed Queueing Networks

- ❑ **Mixed queueing networks:** Open for some workloads and closed for others \Rightarrow Two classes of jobs. **Class** = types of jobs. All jobs of a single class have the same service demands and transition probabilities. Within each class, the jobs are indistinguishable.



Series Networks



- ❑ k M/M/1 queues in series
- ❑ Each individual queue can be analyzed independently of other queues
- ❑ Arrival rate = λ . If μ_i is the service rate for i^{th} server:

Utilization of i^{th} server $\rho_i = \lambda / \mu_i$

Probability of n_i jobs in the i^{th} queue = $(1 - \rho_i) \rho_i^{n_i}$

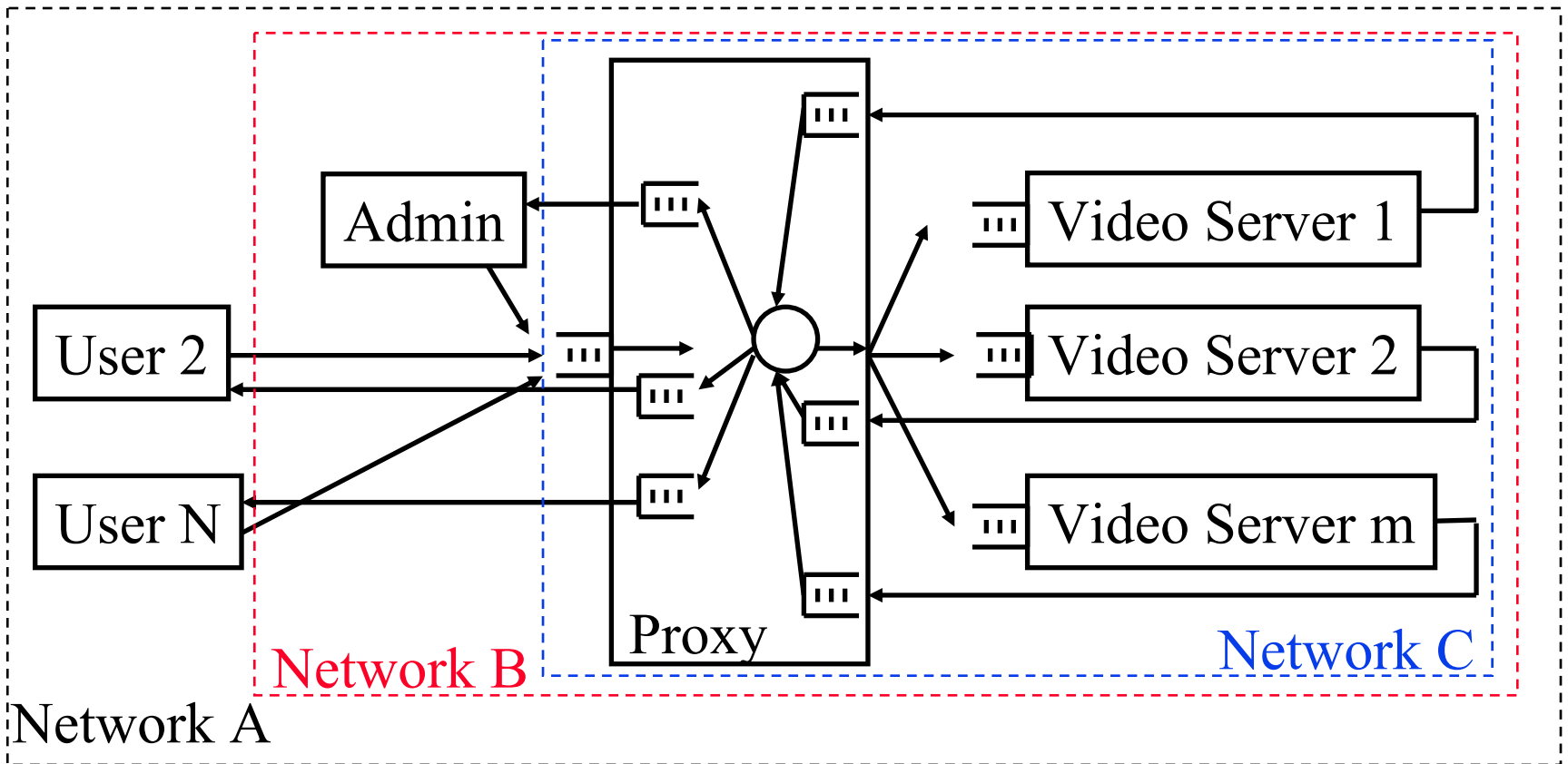
Series Networks (Cont)

- Joint probability of queue lengths:

$$\begin{aligned} & P(n_1, n_2, n_3, \dots, n_M) \\ &= (1 - \rho_1)\rho_1^{n_1} (1 - \rho_2)\rho_2^{n_2} (1 - \rho_3)\rho_3^{n_3} \cdots (1 - \rho_M)\rho_M^{n_M} \\ &= p_1(n_1)p_2(n_2)p_3(n_3) \cdots p_M(n_M) \end{aligned}$$

⇒ product form network

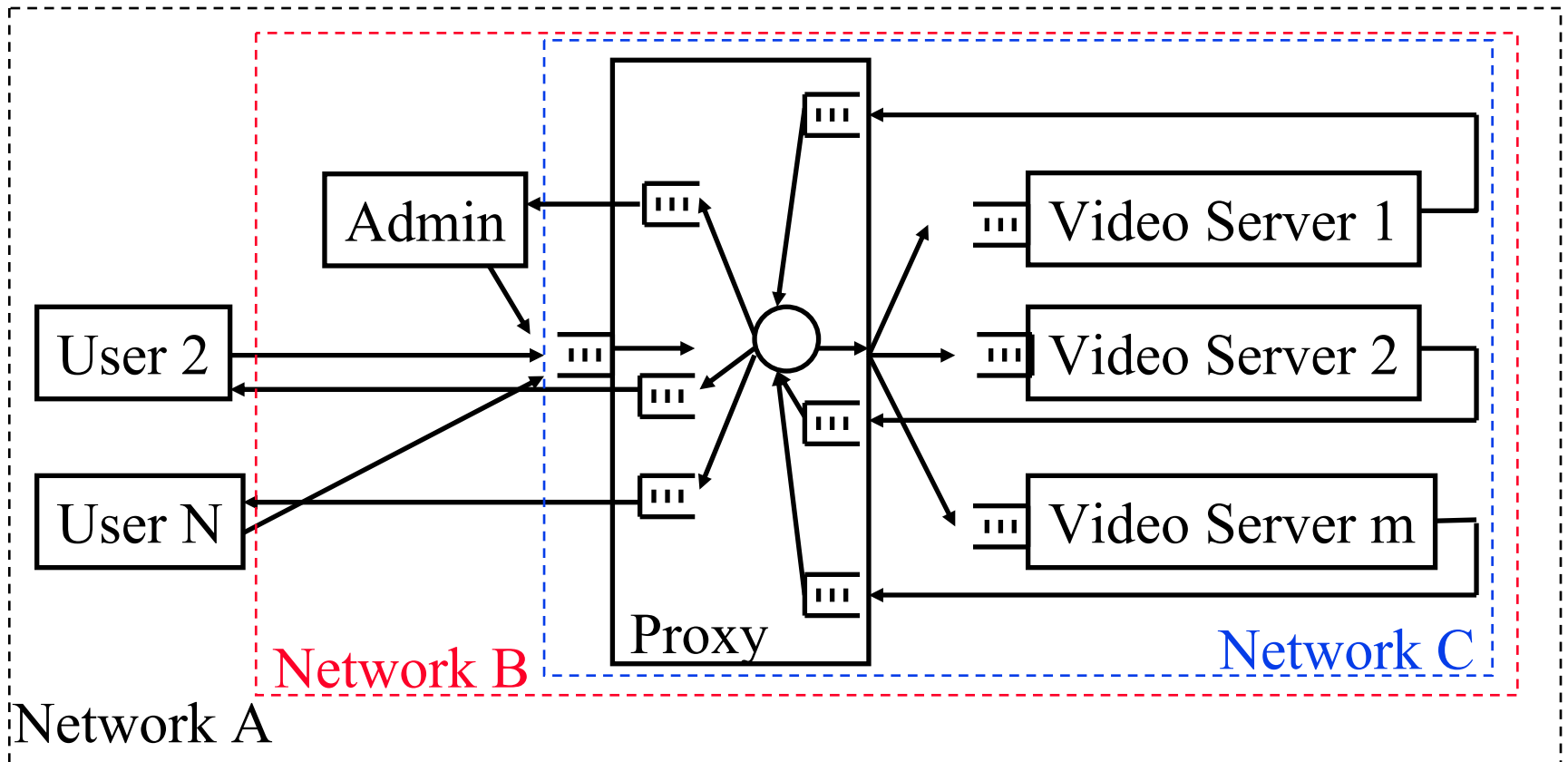
Quiz 32A



Identify open/closed/mixed networks:

- A. Network A is _____
- B. Network B is _____
- C. Network C is _____

Solution to Quiz 32A



Identify open/closed/mixed networks:

- A. Network A is Closed.
- B. Network B is Mixed.
- C. Network C is Open.

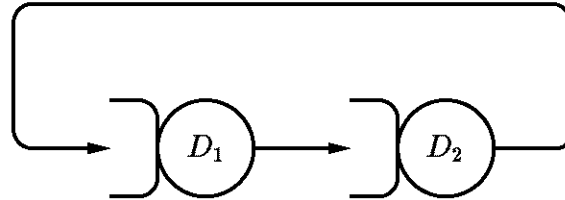
Product-Form Network

- Any queueing network in which:

$$P(n_1, n_2, \dots, n_M) = \frac{1}{G(N)} \prod_{i=1}^M f_i(n_i)$$

- When $f_i(n_i)$ is some function of the number of jobs at the i th facility, $G(N)$ is a normalizing constant and is a function of the total number of jobs in the system.

Example 32.1

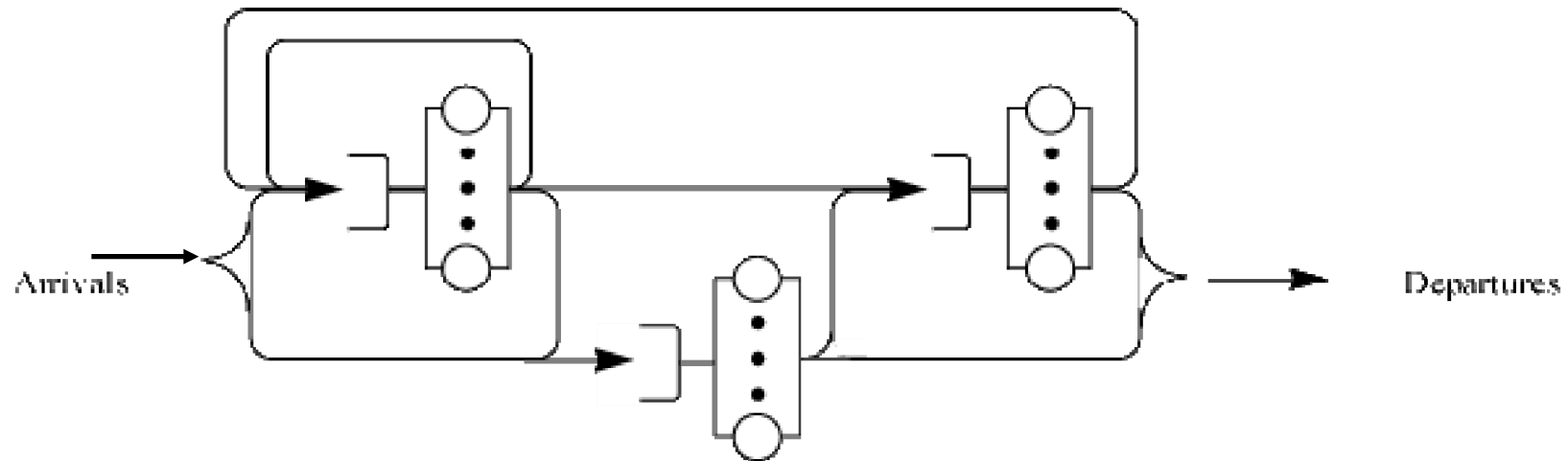


- ❑ Consider a closed system with two queues and N jobs circulating among the queues:
- ❑ Both servers have an exponentially distributed service time. The mean service times are 2 and 3, respectively. The probability of having n_1 jobs in the first queue and $n_2 = N - n_1$ jobs in the second queue can be shown to be:

$$P(n_1, n_2) = \frac{1}{3^{N+1} - 2^{N+1}} (2^{n_1} \times 3^{n_2})$$

- ❑ In this case, the normalizing constant $G(N)$ is $3^{N+1} - 2^{N+1}$.
- ❑ The state probabilities are products of functions of the number of jobs in the queues. Thus, this is a **product form network**.

General Open Network of Queues



- ❑ Product form networks are easier to analyze
- ❑ Jackson (1963) showed that any arbitrary open network of m -server queues with exponentially distributed service times has a product form

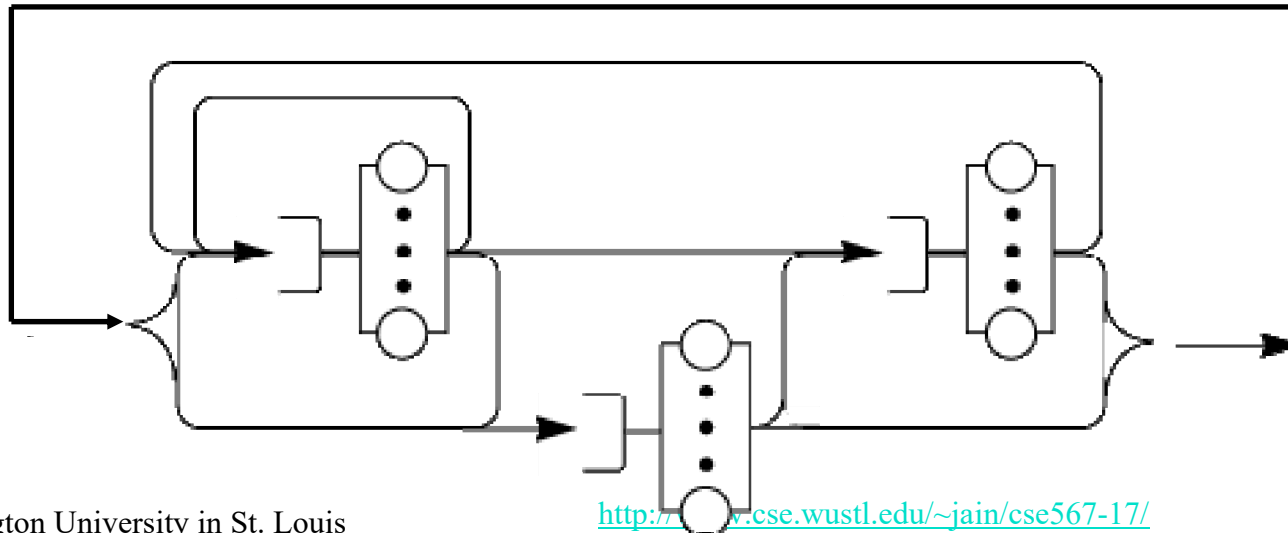
General Open Network of Queues (Cont)

- If all queues are single-server queues, the queue length distribution is:

$$\begin{aligned} &P(n_1, n_2, n_3, \dots, n_M) \\ &= (1 - \rho_1)\rho_1^{n_1} (1 - \rho_2)\rho_2^{n_2} (1 - \rho_3)\rho_3^{n_3} \cdots (1 - \rho_M)\rho_M^{n_M} \\ &= p_1(n_1)p_2(n_2)p_3(n_3) \cdots p_M(n_M) \end{aligned}$$

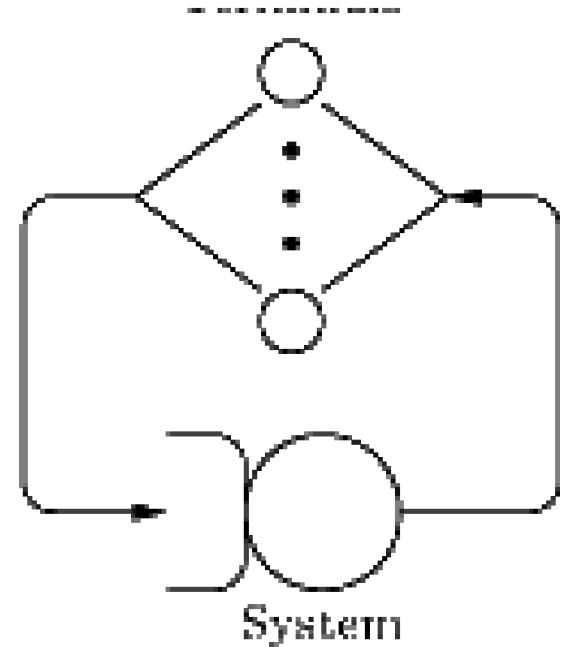
Closed Product-Form Networks

- Gordon and Newell (1967) showed that any arbitrary closed networks of m -server queues with exponentially distributed service times also have a product form solution.
- Baskett, Chandy, Muntz, and Palacios (1975) and then Denning and Buzen (1978) showed that product form solutions exist for an even broader class of networks.
- Note: Internal flows are not Poisson.



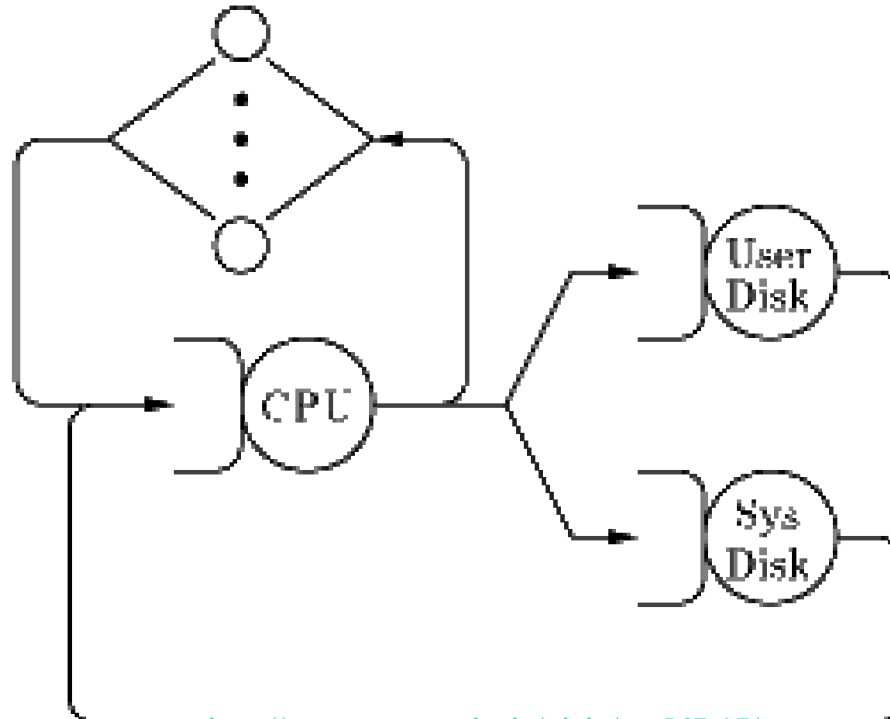
Machine Repairman Model

- ❑ Originally for machine repair shops
- ❑ A number of working machines with a repair facility with one or more servers (repairmen).
- ❑ Whenever a machine breaks down, it is put in the queue for repair and serviced as soon as a repairman is available
- ❑ Scherr (1967) used this model to represent a timesharing system with n terminals.
- ❑ Users sitting at the terminals generate requests (jobs) that are serviced by the system which serves as a repairman.
- ❑ After a job is done, it waits at the user-terminal for a random "think-time" interval before cycling again.



Central Server Model

- ❑ Introduced by Buzen (1973)
- ❑ The CPU is the "central server" that schedules visits to other devices
- ❑ After service at the I/O devices the jobs return to the CPU



Types of Service Centers

Three kinds of devices

1. Fixed-capacity service centers: Service time does not depend upon the number of jobs in the device

For example, the CPU in a system may be modeled as a fixed-capacity service center.

2. Delay centers or infinite server: No queueing. Jobs spend the same amount of time in the device regardless of the number of jobs in it. A group of dedicated terminals is usually modeled as a delay center.

3. Load-dependent service centers: Service rates may depend upon the load or the number of jobs in the device., e.g., $M/M/m$ queue (with $m \geq 2$)

A group of parallel links between two nodes in a computer network is another example

Quiz 32B

- The probability function for jobs in a system with m queues is:

$$P(n_1, n_2, \dots, n_m) = \frac{g(n_1)g(n_2)g(n_{m-1})}{g(n_m)}$$

Is this a product form network? _____

- Identify the type of server:
 - A. Multi-core CPU: _____
 - B. Single-core CPU (No dynamic frequency scaling): _____
 - C. Single-core CPU (with dynamic frequency scaling): _____
 - D. Hard disk drives: _____
 - E. Solid state drives: _____
 - F. Multiple users each handling one window: _____
 - G. A user handling multiple windows: _____

Solution to Quiz 32B

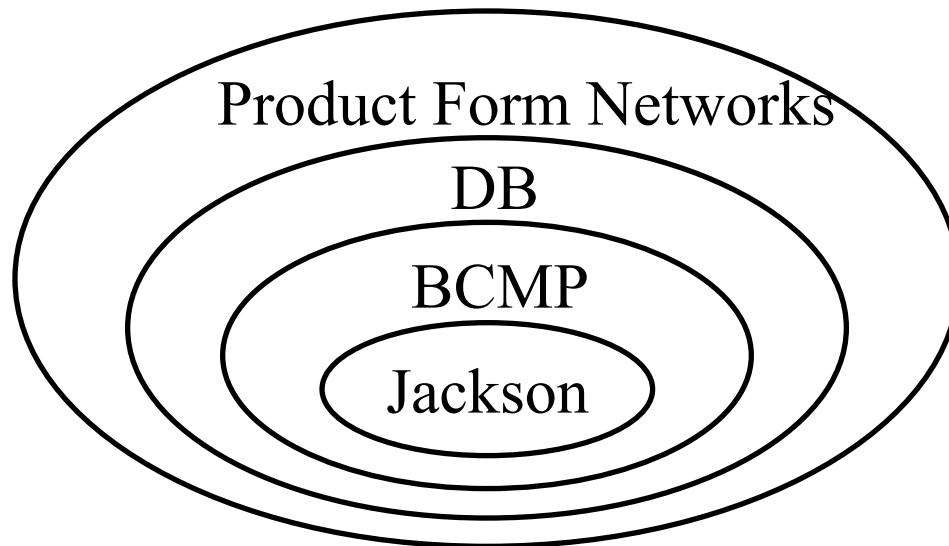
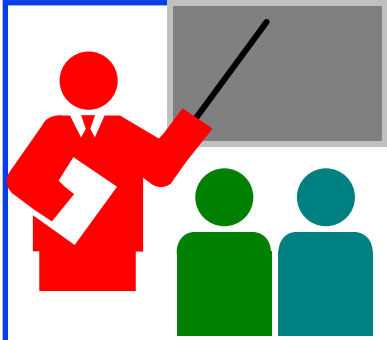
- The probability function for jobs in a system with m queues is:

$$P(n_1, n_2, \dots, n_m) = \frac{g(n_1)g(n_2)g(n_{m-1})}{g(n_m)}$$

Is this a product form network? **YES**

- Identify the type of server:
 - Multi-core CPU: Load dependent
 - Single-core CPU (No dynamic frequency scaling): Fixed Capacity
 - Single-core CPU (with dynamic frequency scaling): Load Dependent
 - Hard disk drives: Load dependent
 - Solid state drives: Fixed capacity
 - Multiple users each handling one window: Delay Center
 - A user handling multiple windows: Fixed capacity

Summary



- ❑ Open, Closed, and Mixed queueing networks
- ❑ Product form networks: Any network in which the system state probability is a product of device state probabilities
- ❑ Jackson: Network of M/M/ m queues.
BCMP: More general conditions
Denning and Buzen: Even more general conditions
- ❑ Service centers: Fixed capacity, delay centers, load dependent

Homework 32

- ❑ In a series network of three routers, the packets arrive at the rate of 100 packets/second. The service rate of the three routers is 250 packets/s, 150 packets/s, and 200 packets/s.
- ❑ Write an expression for the state probability of the system.
- ❑ Calculate the probability of having 2 packets at each of the three routers.



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https://www.youtube.com/playlist?list=PLjGG94etKypJEKjNAa1n_1X0bWWNyZcof

CSE473S: Introduction to Computer Networks (Fall 2011),

https://www.youtube.com/playlist?list=PLjGG94etKypJWOSPMh8Azcgy5e_10TiDw



Wireless and Mobile Networking (Spring 2016),

https://www.youtube.com/playlist?list=PLjGG94etKypKeb0nzyN9tSs_HCd5c4wXF

CSE571S: Network Security (Fall 2011),

<https://www.youtube.com/playlist?list=PLjGG94etKypKvzfVtutHcPFJXumyyg93u>



Video Podcasts of Prof. Raj Jain's Lectures,

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