

# Operational Laws



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- What is an Operational Law?
  1. Utilization Law
  2. Forced Flow Law
  3. Little's Law
  4. General Response Time Law
  5. Interactive Response Time Law
  6. Bottleneck Analysis

## Operational Laws

- Relationships that do not require any assumptions about the distribution of service times or inter-arrival times.
- Identified originally by Buzen (1976) and later extended by Denning and Buzen (1978).
- **Operational** ⇒ Directly measured.
- **Operationally testable assumptions**  
⇒ assumptions that can be verified by measurements.
  - For example, whether number of arrivals is equal to the number of completions?
  - This assumption, called job flow balance, is operationally testable.
  - A set of observed service times is or is not a sequence of independent random variables is not operationally testable.

## Operational Quantities

- Quantities that can be directly measured during a finite observation period. 
- $T$  = Observation interval       $A_i$  = Number of arrivals
- $C_i$  = Number of completions       $B_i$  = Busy time  $B_i$

$$\text{Arrival Rate } \lambda_i = \frac{\text{Number of arrivals}}{\text{Time}} = \frac{A_i}{T}$$

$$\text{Throughput } X_i = \frac{\text{Number of completions}}{\text{Time}} = \frac{C_i}{T}$$

$$\text{Utilization } U_i = \frac{\text{Busy Time}}{\text{Total Time}} = \frac{B_i}{T}$$

$$\text{Mean service time } S_i = \frac{\text{Total time Served}}{\text{Number served}} = \frac{B_i}{C_i}$$

## Utilization Law

$$\begin{aligned} \text{Utilization } U_i &= \frac{\text{Busy Time}}{\text{Total Time}} = \frac{B_i}{T} \\ &= \frac{C_i}{T} \times \frac{B_i}{C_i} = \frac{\text{Completions}}{\text{Time}} \times \frac{\text{Busy Time}}{\text{Completions}} \\ &= \text{Throughput} \times \text{Mean Service Time} = X_i S_i \end{aligned}$$

- This is one of the operational laws
- Operational laws are similar to the elementary laws of motion  
For example,

$$d = \frac{1}{2}at^2$$

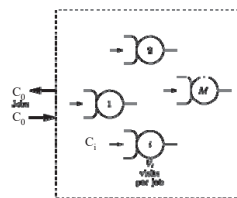
- Notice that distance  $d$ , acceleration  $a$ , and time  $t$  are **operational quantities**. No need to consider them as expected values of random variables or to assume a distribution.

## Example 33.1

- Consider a network gateway at which the packets arrive at a rate of 125 packets per second and the gateway takes an average of two milliseconds to forward them.
- Throughput  $X_i =$  Exit rate = Arrival rate = 125 packets/second
- Service time  $S_i = 0.002$  second
- Utilization  $U_i = X_i S_i = 125 \times 0.002 = 0.25 = 25\%$
- This result is valid for any arrival or service process.  
Even if inter-arrival times and service times are not IID random variables with exponential distribution.

## Forced Flow Law

- Relates the system throughput to individual device throughputs.
- In an open model,  
System throughput  
= # of jobs leaving the system per unit time
- In a closed model, System throughput  
= # of jobs traversing OUT to IN link per unit time.
- If observation period  $T$  is such that  $A_i = C_i$   
 $\Rightarrow$  Device satisfies the assumption of **job flow balance**.
- Each job makes  $V_i$  requests for  $i^{\text{th}}$  device in the system
- $C_i = C_0 V_i$  or  $V_i = C_i / C_0$   $V_i$  is called visit ratio



- System throughput  $X = \frac{\text{Jobs completed}}{\text{Total time}} = \frac{C_0}{T}$

## Forced Flow Law (Cont)

- Throughput of  $i^{\text{th}}$  device:

$$\text{Device Throughput } X_i = \frac{C_i}{T} = \frac{C_i}{C_0} \times \frac{C_0}{T}$$

- In other words:

$$X_i = X V_i$$

- This is the **forced flow law**.

## Bottleneck Device

- Combining the forced flow law and the utilization law, we get:

$$\begin{aligned} \text{Utilization of } i^{\text{th}} \text{ device } U_i &= X_i S_i \\ &= X V_i S_i \\ U_i &= X D_i \end{aligned}$$

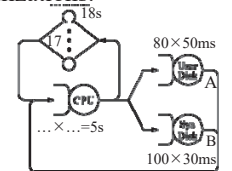
- Here  $D_i = V_i S_i$  is the total service demand on the device for all visits of a job.
- The device with the highest  $D_i$  has the highest utilization and is the **bottleneck device**.

## Example 33.2

- In a timesharing system, accounting log data produced the following profile for user programs.
  - Each program requires five seconds of CPU time, makes 80 I/O requests to the disk A and 100 I/O requests to disk B.
  - Average think-time of the users was 18 seconds.
  - From the device specifications, it was determined that disk A takes 50 milliseconds to satisfy an I/O request and the disk B takes 30 milliseconds per request.
  - With 17 active terminals, disk A throughput was observed to be 15.70 I/O requests per second.

- We want to find the system throughput and device utilizations.

$$\begin{aligned} D_{CPU} &= 5 \text{ seconds} & V_A &= 80, \\ V_B &= 100, & Z &= 18 \text{ seconds}, \\ S_A &= 0.050 \text{ seconds}, & S_B &= 0.030 \text{ seconds}, \\ N &= 17, \text{ and} & X_A &= 15.70 \text{ jobs/second} \end{aligned}$$



## Example 33.2 (Cont)

$$\begin{aligned} D_{CPU} &= 5 \text{ seconds} & V_A &= 80, \\ V_B &= 100, & Z &= 18 \text{ seconds}, \\ S_A &= 0.050 \text{ seconds}, & S_B &= 0.030 \text{ seconds}, \\ N &= 17, \text{ and} & X_A &= 15.70 \text{ jobs/second} \end{aligned}$$

- Since the jobs must visit the CPU before going to the disks or terminals, the CPU visit ratio is:  $V_{CPU} = V_A + V_B + 1 = 181$

$$D_{CPU} = 5 \text{ seconds}$$

$$D_A = S_A V_A = 0.050 \times 80 = 4 \text{ seconds}$$

$$D_B = S_B V_B = 0.030 \times 100 = 3 \text{ seconds}$$

- Using the forced flow law, the throughputs are:

$$X = \frac{X_A}{V_A} = \frac{15.70}{80} = 0.1963 \text{ jobs/second}$$

$$X_{CPU} = X V_{CPU} = 0.1963 \times 181$$

$$= 35.48 \text{ requests/second}$$

$$X_B = X V_B = 0.1963 \times 100$$

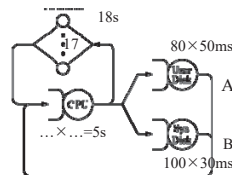
$$= 19.6 \text{ requests/second}$$

- Using the utilization law, the device utilizations are:

$$U_{CPU} = X D_{CPU} = 0.1963 \times 5 = 98\%$$

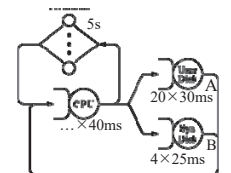
$$U_A = X D_A = 0.1963 \times 4 = 78.4\%$$

$$U_B = X D_B = 0.1963 \times 3 = 58.8\%$$



## Homework 33A

- The visit ratios and service time per visit for a system are as shown:
- For each device what is the total service demand:
  - CPU:  $V_i = \underline{\hspace{2cm}}, S_i = \underline{\hspace{2cm}}, D_i = \underline{\hspace{2cm}}$
  - Disk A:  $V_i = \underline{\hspace{2cm}}, S_i = \underline{\hspace{2cm}}, D_i = \underline{\hspace{2cm}}$
  - Disk B:  $V_i = \underline{\hspace{2cm}}, S_i = \underline{\hspace{2cm}}, D_i = \underline{\hspace{2cm}}$
  - Terminals:  $V_i = \underline{\hspace{2cm}}, S_i = \underline{\hspace{2cm}}, D_i = \underline{\hspace{2cm}}$
- If disk A utilization is 50%, what's the utilization of CPU and Disk B?
  - $X_A = U_A / D_A = \underline{\hspace{2cm}}$
  - $U_{CPU} = X D_{CPU} = \underline{\hspace{2cm}}$
  - $U_B = X D_B = \underline{\hspace{2cm}}$
- What is the bottleneck device?  $\underline{\hspace{2cm}}$



Key:  $U_i = X_i S_i = X D_i, D_i = S_i V_i, X = X_i / V_i$

## Transition Probabilities

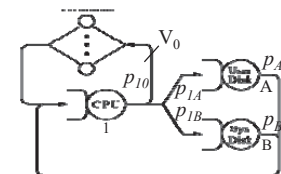
- $p_{ij}$  = Probability of a job moving to  $j^{\text{th}}$  queue after service completion at  $i^{\text{th}}$  queue
- Visit ratios and transition probabilities are equivalent in the sense that given one we can always find the other.
- In a system with job flow balance:  $C_j = \sum_{i=0}^M C_i p_{ij}$   
 $i=0 \Rightarrow$  visits to the outside link
- $p_{i0}$  = Probability of a job exiting from the system after completion of service at  $i^{\text{th}}$  device
- Dividing by  $C_0$  we get:  $V_j = \sum_{i=0}^M V_i p_{ij}$

## Transition Probabilities (Cont)

- Since each visit to the outside link is defined as the completion of the job, we have:  $V_0 = 1$
- These are called **visit ratio equations**
- In central server models, after completion of service at every queue, the jobs always move back to the CPU queue:

$$p_{i1} = 1 \quad \forall i \neq 1$$

$$p_{ij} = 0 \quad \forall i, j \neq 1$$



## Transition Probabilities (Cont)

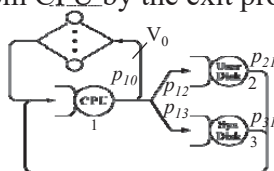
- The above probabilities apply to exit and entrances from the system ( $i=0$ ), also. Therefore, the visit ratio equations become:

$$1 = V_1 p_{10} \quad \Rightarrow \quad V_1 = \frac{1}{p_{10}}$$

$$V_1 = 1 + V_2 + V_3 + \dots + V_M$$

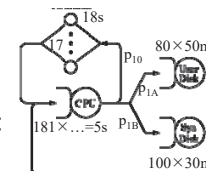
$$V_j = V_1 p_{1j} = \frac{p_{1j}}{p_{10}} \quad j = 2, 3, \dots, M$$

- Thus, we can find the visit ratios by dividing the probability  $p_{1j}$  of moving to  $j^{\text{th}}$  queue from CPU by the exit probability  $p_{10}$ .



## Example 33.3

- Consider the queueing network:



- The visit ratios are  $V_A=80$ ,  $V_B=100$ , and  $V_{CPU}=181$ .
- After completion of service at the CPU the probabilities of the job moving to disk A, disk B, or terminals are  $80/181$ ,  $100/181$ , and  $1/181$ , respectively. Thus, the transition probabilities are  $p_{1A}=0.4420$ ,  $p_{1B}=0.5525$ , and  $p_{10}=0.005525$ .
- Given the transition probabilities, we can find the visit ratios by dividing these probabilities by the exit probability (0.005525):

$$V_A = \frac{p_{1A}}{p_{10}} = \frac{0.4420}{0.005525} = 80$$

$$V_B = \frac{p_{1B}}{p_{10}} = \frac{0.5525}{0.005525} = 100$$

$$V_{CPU} = 1 + V_A + V_B = 1 + 80 + 100 = 181$$

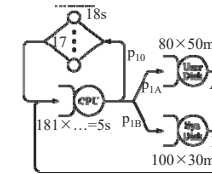
## Little's Law

Mean number in the device  
 = Arrival rate  $\times$  Mean time in the device  
 $Q_i = \lambda_i R_i$

- If the job flow is balanced, the arrival rate is equal to the throughput and we can write:

$$Q_i = X_i R_i$$

## Example 33.4



- The average queue length in the computer system of Example 33.2 was observed to be: 8.88, 3.19, and 1.40 jobs at the CPU, disk A, and disk B, respectively. What were the response times of these devices?
- In Example 33.2, the device throughputs were determined to be:  $X_{CPU} = 35.48$ ,  $X_A = 15.70$ , and  $X_B = 19.6$
- The new information given in this example is:  
 $Q_{CPU} = 8.88$ ,  $Q_A = 3.19$ , and  $Q_B = 1.40$
- Using Little's law, the device response times are:  
 $R_{CPU} = Q_{CPU}/X_{CPU} = 8.88/35.48 = 0.250$  seconds  
 $R_A = Q_A/X_A = 3.19/15.70 = 0.203$  seconds  
 $R_B = Q_B/X_B = 1.40/19.6 = 0.071$  seconds

## General Response Time Law

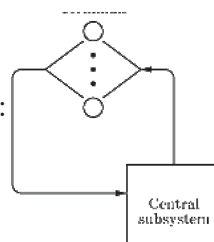
- There is one terminal per user and the rest of the system is shared by all users.
- Applying Little's law to the central subsystem:

$$Q = X R$$

- Here,
- $Q$  = Total number of jobs in the system
- $R$  = system response time
- $X$  = system throughput

$$Q = Q_1 + Q_2 + \dots + Q_M$$

$$X R = X_1 R_1 + X_2 R_2 + \dots + X_M R_M$$



## General Response Time Law (Cont)

$$X R = X_1 R_1 + X_2 R_2 + \dots + X_M R_M$$

- Dividing both sides by  $X$  and using forced flow law:

$$R = V_1 R_1 + V_2 R_2 + \dots + V_M R_M$$

- or,

$$R = \sum_{i=1}^M R_i V_i$$

- This is called the **general response time law**.

## Example 33.5

- Let us compute the response time for the timesharing system of Example 33.4

- For this system:

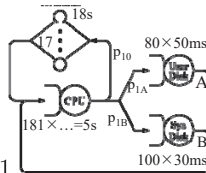
$$V_{CPU} = 181, V_A = 80, \text{ and } V_B = 100$$

$$R_{CPU} = 0.250, R_A = 0.203, \text{ and } R_B = 0.071$$

- The system response time is:

$$\begin{aligned} R &= R_{CPU}V_{CPU} + R_A V_A + R_B V_B \\ &= 0.250 \times 181 + 0.203 \times 80 + 0.071 \times 100 \\ &= 68.6 \end{aligned}$$

- The system response time is 68.6 seconds.



## Homework 33B

- The transition probabilities of jobs exiting CPU and device service times are as shown.

- Find the visit ratios:

$$V_A = p_{1A}/p_{10} = \underline{\hspace{2cm}}$$

$$V_B = p_{1B}/p_{10} = \underline{\hspace{2cm}}$$

$$V_{CPU} = 1 + V_A + V_B = \underline{\hspace{2cm}}$$

- The queue lengths at CPU, disk A, and disk B was observed to be 6, 3, and 1, respectively. The system throughput is 1 jobs/sec. What is the system response time?

$$R_{CPU} = Q_{CPU}/X_{CPU} = Q_{CPU}/(XV_{CPU}) = \underline{\hspace{2cm}}$$

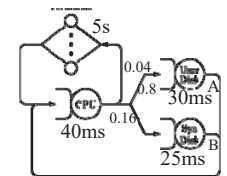
$$R_A = Q_A/(X_A) = \underline{\hspace{2cm}}$$

$$R_B = Q_B/(X_B) = \underline{\hspace{2cm}}$$

$$R = R_{CPU}V_{CPU} + R_A V_A + R_B V_B = \underline{\hspace{2cm}}$$

$$\text{Check: } Q = X R \underline{\hspace{2cm}}$$

$$\text{Key: } U_i = X_i S_i = X D_i, D_i = S_i V_i, X = X_i/V_i, Q_i = X_i R_i, R = \sum_{i=1}^M R_i V_i$$



## Interactive Response Time Law

- If  $Z$  = think-time,  $R$  = Response time

- The total cycle time of requests is  $R+Z$

- Each user generates about  $T/(R+Z)$  requests in  $T$

- If there are  $N$  users:

$$\begin{aligned} \text{System throughput } X &= \text{Total \# of requests/Total time} \\ &= N(T/(R+Z))/T \\ &= N/(R+Z) \end{aligned}$$

or

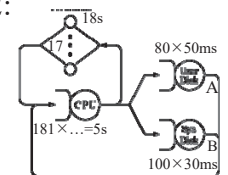
$$R = (N/X) - Z$$

- This is the interactive response time law

## Example 33.6

- For the timesharing system of Example 33.2:

$$X = 0.1963, N = 17, \text{ and } Z = 18$$



The response time can be calculated as follows:

$$R = \frac{N}{X} - Z = \frac{17}{0.1963} - 18 = 86.6 - 18 = 68.6 \text{ seconds}$$

- This is the same as that obtained earlier in Example 33.5.

## Review of Operational Laws

### Operational quantities:

Can be measured by operations personnel

$V_i$  = # of visits per job to device  $i$

$S_i$  = Service time per job at device  $i$

$D_i$  = Total service demands per job at device  $i = S_i V_i$

$X_i$  = Throughput of device  $i$

$X$  = Throughput of the system

$Z$  = User think time

$N$  = Number of users in a time shared system

### Operational assumptions:

That can be easily validated.

# Input = # output (**flow balance**) can be validated

Distributions and independence can not be validated.

### Operational Laws:

Relationships between operational quantities

These apply regardless of distribution, burstiness, arrival patterns.

The only assumption is flow balance.

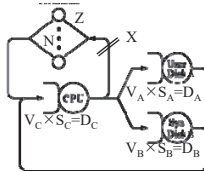
1. Utilization Law:  $U = X_i S_i = X D_i$

2. Forced Flow Law:  $X_i = X V_i$

3. Little's Law:  $Q_i = X_i R_i$

4. General Response Time Law:  $R = \sum R_i V_i$

5. Interactive Response Time Law:  $R = N/X - Z$



## Example

### Operational quantities:

Can be measured by operations personnel

$V_i$  = # of visits per job to device  $i = 181, 80, 100$

$S_i$  = Service time per job at device  $i = 27.6ms, 50ms, 30ms$

$D_i$  = Total service demands per job at device  $i = S_i V_i = 5s, 4s, 3s$

$Z$  = User think time = 18s

$N$  = Number of users in a time shared system = 12

### Operational Laws:

Given  $U_A = 75\%$ ,  $Q_A = 2.41$ ,  $Q_B = 1.21$ ,  $Q_C = 5$

1. Utilization Law:  $U = X_i S_i = X D_i$

$X = U_A / D_A = 0.75 / 4 = 0.188$  jobs/s

$U_C = X \times D_C = 0.188 \times 5 = 0.939$

$U_B = X \times D_B = 0.188 \times 3 = 0.563$

2. Forced Flow Law:  $X_i = X V_i$

$X_A = X \times 80 = 0.188 \times 80 = 15$  jobs/s

$X_B = X \times 100 = 0.188 \times 100 = 18.8$  jobs/s

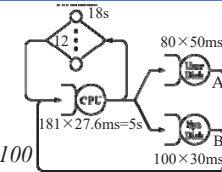
$X_C = X \times 181 = 0.188 \times 181 = 34$  jobs/s

3. Little's Law:  $Q_i = X_i R_i$

$R_A = Q_A / X_A = 2.41 / 15 = 0.161$ ,  $R_B = 1.21 / 18.8 = 0.064$ ,  $R_C = 5 / 34 = 0.147$

4. General Response Time Law:  $R = \sum R_i V_i = 0.161 \times 80 + 0.064 \times 100 + 0.147 \times 181 = 45.89s$

5. Interactive Response Time Law:  $R = N/X - Z = 12 / 0.188 - 18 = 45.83s$



## Homework 33C

### Operational quantities:

Can be measured by operations personnel

$V_i$  = # of visits per job to device  $i = 91, 50, 40$

$S_i$  = Service time per job at device  $i = 0.044s, 0.040s, 0.025s$

$Z$  = User think time = 5s     $N$  = Number of users = 6

### Operational Laws:

Given  $U_A = 48\%$ ,  $R_A = 0.0705s$ ,  $R_B = 0.0323s$ ,  $R_C = 0.1668s$

1.  $D_i$  = Total service demands per job at device  $i = S_i V_i$   
 $D_C = S_C V_C = \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_$ ,  $D_A = \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_$ ,  $D_B = \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_$

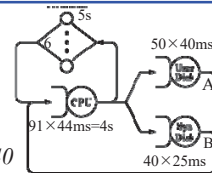
2. Utilization Law:  $U = X_i S_i = X D_i$   
 $X = U_A / D_A = \_\_\_\_\_\_ / \_\_\_\_\_\_ = \_\_\_\_\_\_ \text{ jobs/s}$   
 $U_C = X \times D_C = \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_$   
 $U_B = X \times D_B = \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_$

3. Forced Flow Law:  $X_i = X V_i$   
 $X_A = X \times V_A = \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_ \text{ jobs/s}$   
 $X_B = X \times V_B = \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_ \text{ jobs/s}$   
 $X_C = X \times V_C = \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_ \text{ jobs/s}$

4. Little's Law:  $Q_i = X_i R_i$   
 $Q_A = \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_$ ,  $Q_B = \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_$ ,  $Q_C = \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_$

5. General Response Time Law:  $R = \sum R_i V_i$   
 $= \_\_\_\_\_\_ \times \_\_\_\_\_\_ + \_\_\_\_\_\_ \times \_\_\_\_\_\_ + \_\_\_\_\_\_ \times \_\_\_\_\_\_ = \_\_\_\_\_\_ \text{ s}$

6. Interactive Response Time Law:  $R = N/X - Z = \_\_\_\_\_\_ / \_\_\_\_\_\_ - \_\_\_\_\_\_ = \_\_\_\_\_\_ \text{ s}$



## Bottleneck Analysis

### From forced flow law:

$$U_i \propto D_i$$

The device with the highest total service demand  $D_i$  has the highest utilization and is called the bottleneck device.

Note: Delay centers can have utilizations more than one without any stability problems. Therefore, delay centers cannot be a bottleneck device.

Only queuing centers used in computing  $D_{max}$ .

The bottleneck device is the key limiting factor in achieving higher throughput.

## Bottleneck Analysis (Cont)

- Improving the bottleneck device will provide the highest payoff in terms of system throughput.
- Improving other devices will have little effect on the system performance.
- Identifying the bottleneck device should be the first step in any performance improvement project.

## Asymptotic Bounds

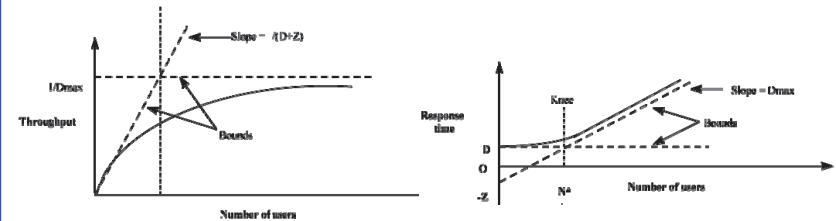
- Throughput and response times of the system are bound as follows:

$$X(N) \leq \min\left\{\frac{1}{D_{max}}, \frac{N}{D+Z}\right\}$$

and

$$R(N) \geq \max\{D, ND_{max} - Z\}$$

- Here,  $D = \sum D_i$  is the sum of total service demands on all devices except terminals.



## Asymptotic Bounds: Proof

- The asymptotic bounds are based on the following observations:
  - The utilization of any device cannot exceed one. This puts a limit on the maximum obtainable throughput.
  - The response time of the system with  $N$  users cannot be less than a system with just one user. This puts a limit on the minimum response time.
  - The interactive response time formula can be used to convert the bound on throughput to that on response time and vice versa.

## Proof (Cont)

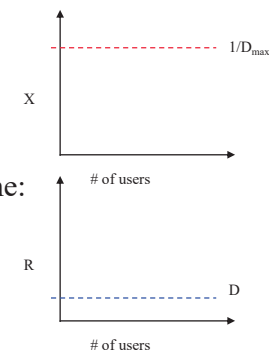
- For the bottleneck device  $b$ :

$$U_b = X D_{max}$$

Since  $U_b$  cannot be more than one:

$$X D_{max} \leq 1$$

$$X \leq \frac{1}{D_{max}}$$



- With just one job in the system, there is no queueing and the system response time is simply the sum of the service demands:

$$R(1) = D_1 + D_2 + \dots + D_M = D$$

With more than one user there may be some queueing and so the response time will be higher. That is:

$$R(N) \geq D$$



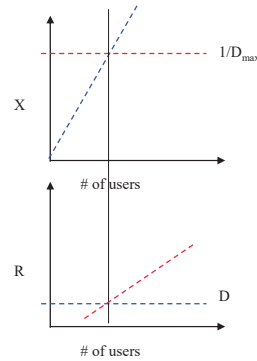
## Proof (Cont)

3. Applying the interactive response time law to the bounds:

$$R = (N/X) - Z$$

$$R(N) = \frac{N}{X(N)} - Z \geq ND_{max} - Z$$

$$X(N) = \frac{N}{R(N) + Z} \leq \frac{N}{D + Z}$$



## Optimal Operating Point

□ The number of jobs  $N^*$  at the knee is given by:

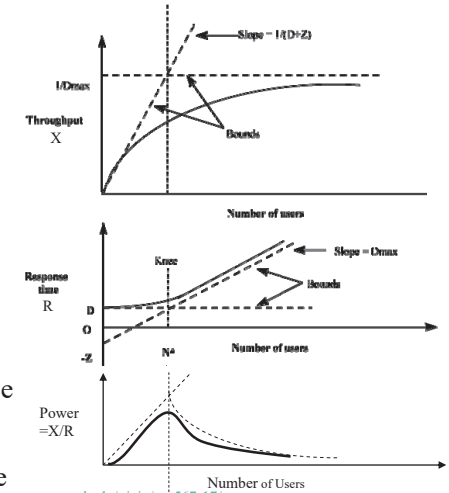
$$D = N^* D_{max} - Z$$

$$N^* = \frac{D + Z}{D_{max}}$$

□ If the number of jobs is more than  $N^*$ , then we can say with certainty that there is queuing somewhere in the system.

□ The asymptotic bounds can be easily explained to people who do not have any background in queuing theory or performance analysis.

□ Control Strategy:  
Increase  $N$  iff  $dP/dN$  is positive



## Example 33.7

□ For the timesharing system of Example 33.2:

$$D_{CPU} = 5, D_A = 4, D_B = 3, Z = 18$$

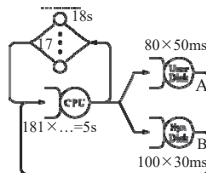
$$D = D_{CPU} + D_A + D_B = 5 + 4 + 3 = 12$$

$$D_{max} = D_{CPU} = 5$$

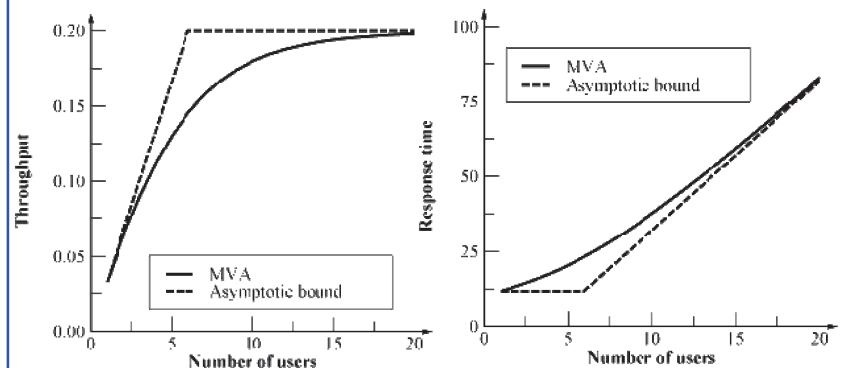
□ The asymptotic bounds are:

$$X(N) \leq \min \left\{ \frac{N}{D + Z}, \frac{1}{D_{max}} \right\} = \min \left\{ \frac{N}{30}, \frac{1}{5} \right\}$$

$$R(N) \geq \max \{ D, ND_{max} - Z \} = \max \{ 12, 5N - 18 \}$$



## Example 33.7: Asymptotic Bounds



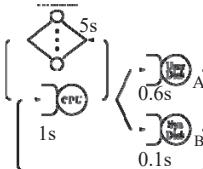
□ The knee occurs at:

$$12 = 5N^* - 18$$

$$N^* = \frac{12 + 18}{5} = \frac{30}{5} = 6$$

## Homework 33D

- ❑ The total demands on various devices are as shown.
- ❑ What is the minimum response time?  
 $R = D = D_{CPU} + D_A + D_B = \underline{\hspace{2cm}}$
- ❑ What is the bottleneck device?  $\underline{\hspace{2cm}}$
- ❑ What is the maximum possible utilization of disk B?  
 $U_B = \underline{\hspace{2cm}}$
- ❑ What is the maximum possible throughput?  $X = \underline{\hspace{2cm}}$
- ❑ What is the upper bound on throughput with  $N$  users?  
 $\underline{\hspace{2cm}}$



- ❑ What is the lower bound on response time with  $N$  users?  
 $\underline{\hspace{2cm}}$
- ❑ What is the knee capacity of this system?  $\underline{\hspace{2cm}}$

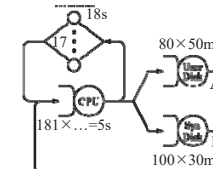
Key:  $R \geq \max\{D, ND_{max} - Z\}$ ,  $X \leq \min\{1/D_{max}, N/(D+Z)\}$

## Example 33.8

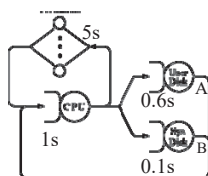
- ❑ How many terminals can be supported on the timesharing system of Example 33.2 if the response time has to be kept below 100 seconds?
- ❑ Using the asymptotic bounds on the response time we get:

$$R(N) \geq \max\{12, 5N - 18\}$$

- ❑ The response time will be more than 100, if:  $5N - 18 \geq 100$
- ❑ That is, if:  $N \geq 23.6$  the response time is bound to be more than 100. Thus, the system cannot support more than 23 users if a response time of less than 100 is required.



## Homework 33E



- ❑ For this system, which device would be the bottleneck if:
- ❑ The CPU is replaced by another unit that is twice as fast?  $\underline{\hspace{2cm}}$
- ❑ Disk A is replaced by another unit that is twice as slow?  $\underline{\hspace{2cm}}$
- ❑ Disk B is replaced by another unit that is twice as slow?  $\underline{\hspace{2cm}}$
- ❑ The memory size is reduced so that the jobs make 25 times more visits to disk B due to increased page faults?  $\underline{\hspace{2cm}}$



## Summary

Utilization Law:	$U_i = X_i S_i = X D_i$
Forced Flow Law:	$X_i = X V_i$
Little's Law:	$Q_i = X_i R_i$
General Response Time Law:	$R = \sum_{i=1}^M R_i V_i$
Interactive Response Time Law:	$R = \frac{N}{X} - Z$
Asymptotic Bounds:	$R \geq \max\{D, ND_{max} - Z\}$
	$X \leq \min\{1/D_{max}, N/(D+Z)\}$

### ❑ Symbols:

$D$	=	Sum of service demands on all devices = $\sum_i D_i$
$D_i$	=	Total service demand per job for $i$ th device = $S_i V_i$
$D_{max}$	=	Service demand on the bottleneck device = $\max_i\{D_i\}$
$N$	=	Number of jobs in the system
$Q_i$	=	Number in the $i$ th device
$R$	=	System response time
$R_i$	=	Response time per visit to the $i$ th device
$S_i$	=	Service time per visit to the $i$ th device
$U_i$	=	Utilization of $i$ th device
$V_i$	=	Number of visits per job to the $i$ th device
$X$	=	System throughput
$X_i$	=	Throughput of the $i$ th device
$Z$	=	Think time


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