Pseudorandom Number Generation and Stream Ciphers

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Audio/Video recordings of this lecture are available at:

http://www.cse.wustl.edu/~jain/cse571-14/

1. Principles of Pseudorandom Number Generation
2. Pseudorandom number generators
3. Pseudorandom number generation using a block cipher
4. Stream Cipher
5. RC4

These slides are based on Lawrie Brown's slides supplied with William Stalling's book "Cryptography and Network Security: Principles and Practice," 6th Ed, 2013.

Pseudo Random Numbers

- Many uses of **random numbers** in cryptography
  - nonces in authentication protocols to prevent replay
  - keystream for a one-time pad
- These values should be
  - statistically random, uniform distribution, independent
  - unpredictability of future values from previous values
- True random numbers provide this
- Psuedo $\Rightarrow$ Deterministic, reproducible, generated by a formula

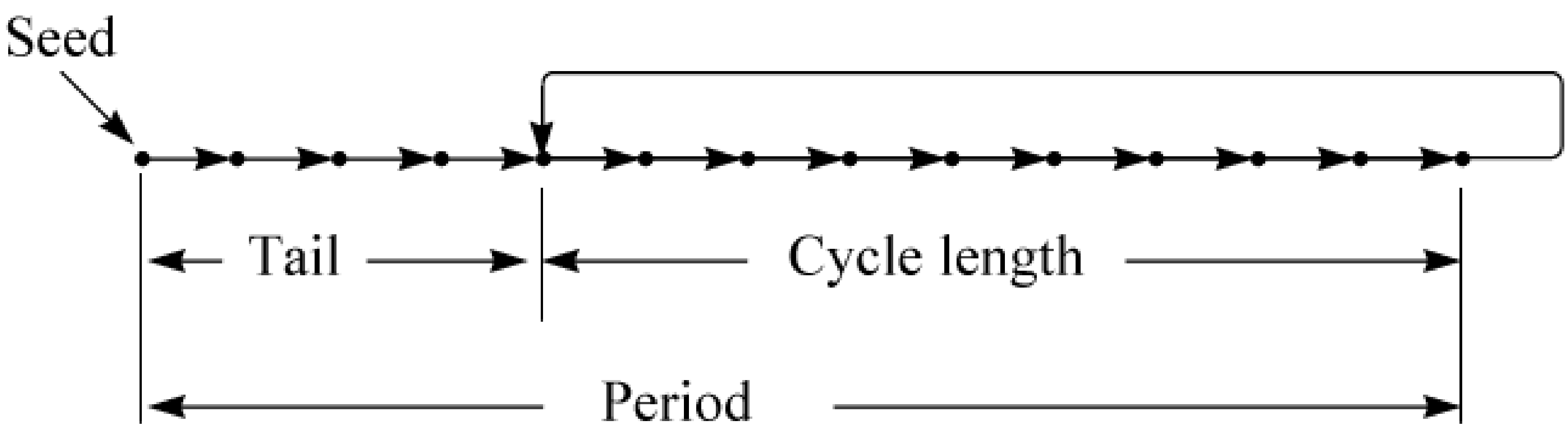
A Sample Generator

$$x_n = f(x_{n-1}, x_{n-2}, \ldots)$$

- For example,
  $$x_n = 5x_{n-1} + 1 \mod 16$$
- Starting with x_0=5:
  $$x_1 = 5(5) + 1 \mod 16 = 26 \mod 16 = 10$$
- The first 32 numbers obtained by the above procedure 10, 3, 0, 1, 6, 15, 12, 13, 2, 11, 8, 9, 14, 7, 4, 5 10, 3, 0, 1, 6, 15, 12, 13, 2, 11, 8, 9, 14, 7, 4, 5.
- By dividing x's by 16:
  0.6250, 0.1875, 0.0000, 0.0625, 0.3750, 0.9375, 0.7500, 0.8125, 0.1250, 0.6875, 0.5000, 0.5625, 0.8750, 0.4375, 0.2500, 0.3125, 0.6250, 0.1875, 0.0000, 0.0625, 0.3750, 0.9375, 0.7500, 0.8125, 0.1250, 0.6875, 0.5000, 0.5625, 0.8750, 0.4375, 0.2500, 0.3125.

Terminology

- **Seed** = x_0
- **Pseudo-Random**: Deterministic yet would pass randomness tests
- Fully Random: Not repeatable
- **Cycle length**, **Tail**, **Period**



Linear-Congruential Generators

- Discovered by D. H. Lehmer in 1951
- The residues of successive powers of a number have good randomness properties.

$$x_n = a^n \bmod m$$

Equivalently,

$$x_n = a x_{n-1} \bmod m$$

a = multiplier

m = modulus

Linear-Congruential Generators (Cont)

- Lehmer's choices: $a = 23$ and $m = 10^8+1$
- Good for ENIAC, an 8-digit decimal machine.
- Generalization:

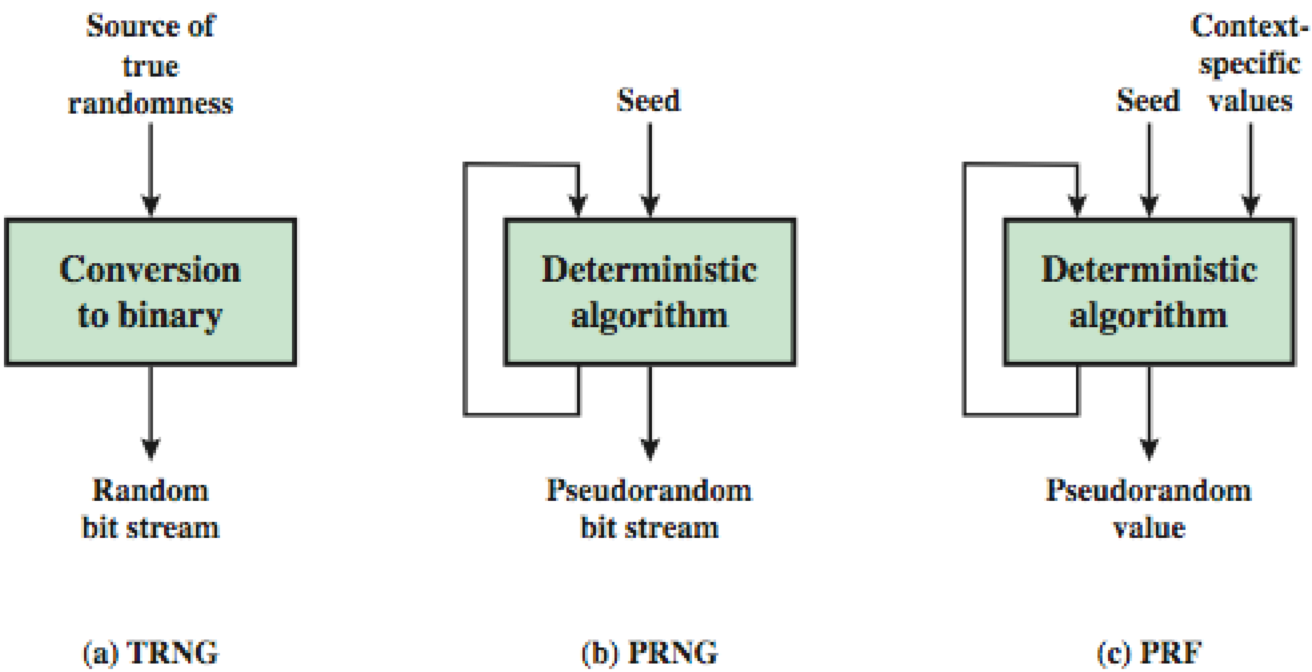
$$x_n = ax_{n-1} + b \bmod m$$

- Can be analyzed easily using the theory of congruences
  $\Rightarrow$ Mixed Linear-Congruential Generators
  or Linear-Congruential Generators (LCG)
- Mixed = both multiplication by a and addition of b

Blum Blum Shub Generator

- Use least significant bit from iterative equation:
  - $x_i = x_{i-1}^2 \bmod n$
  - where $n=p.q$, and primes $p,q=3 \bmod 4$
- Unpredictable, passes **next-bit** test
- Security rests on difficulty of factoring N
- Is unpredictable given any run of bits
- Slow, since very large numbers must be used
- Too slow for cipher use, good for key generation

Random & Pseudorandom Number Generators



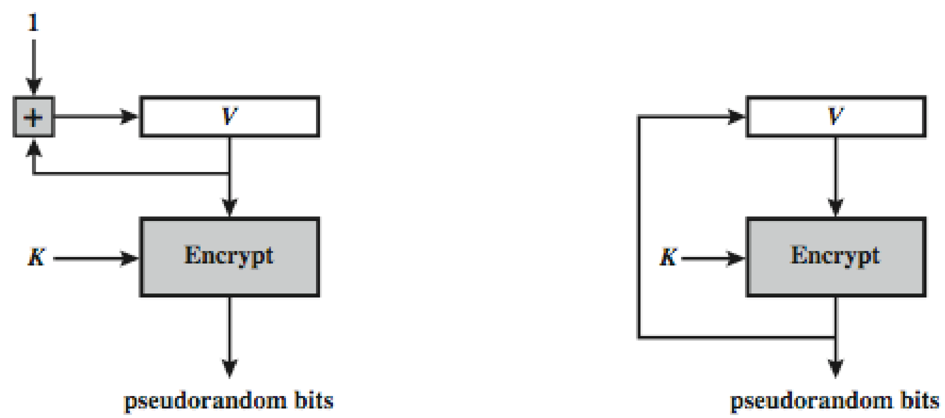
Using Block Ciphers as PRNGs

- Can use a block cipher to generate random numbers for cryptographic applications,
- For creating session keys from master key
- CTR

$$X_i = E_K[V_i]$$

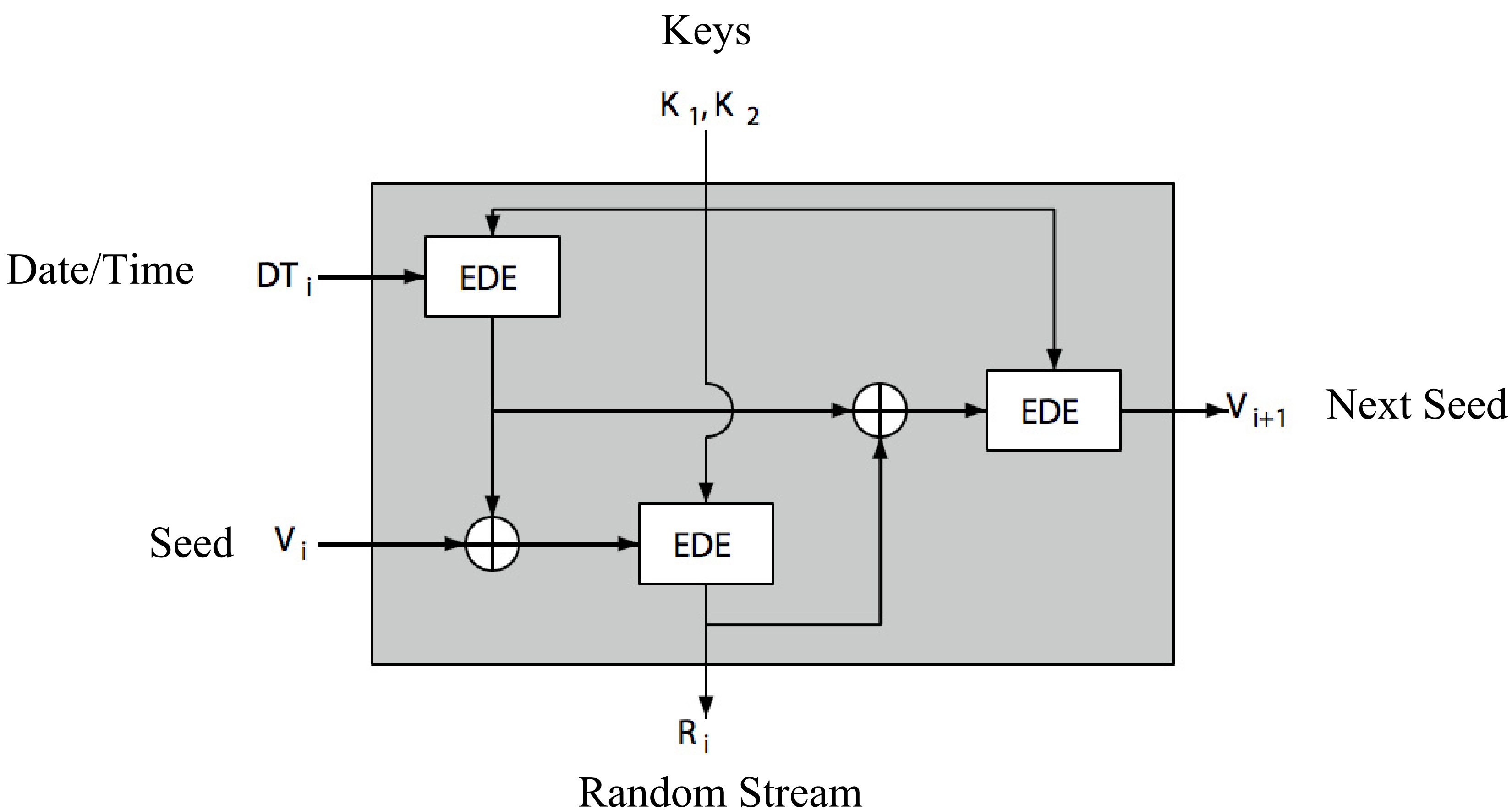
- OFB

$$X_i = E_K[X_{i-1}]$$



(a) CTR Mode (b) OFB Mode

ANSI X9.17 PRG



Natural Random Noise

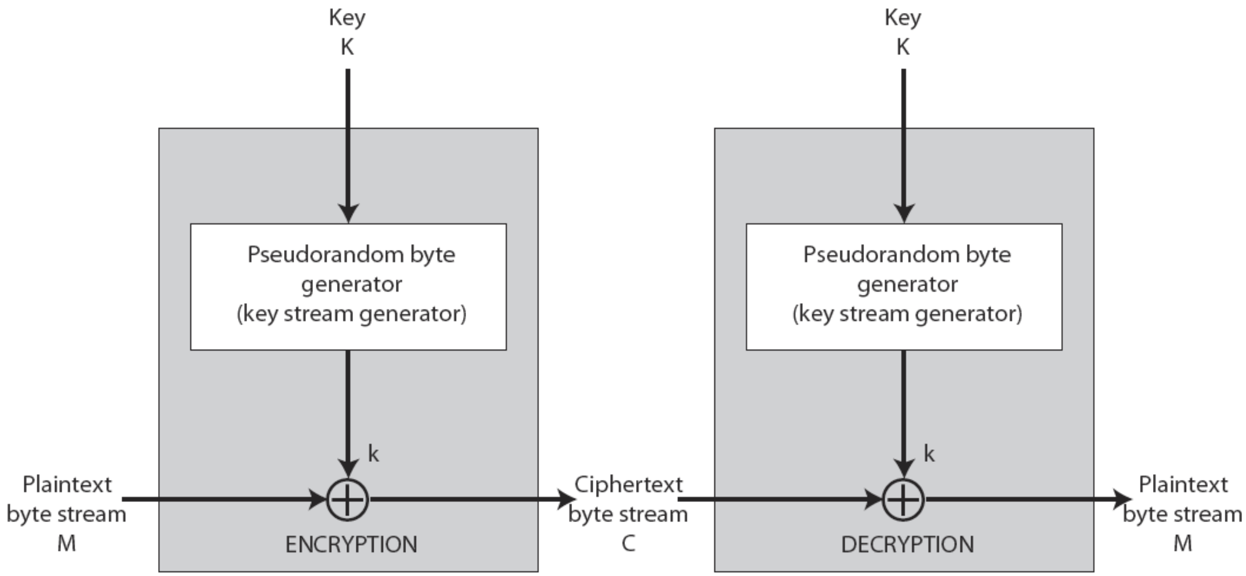
- Best source is natural randomness in real world
- Find a regular but random event and monitor
- Do generally need special h/w to do this
  - E.g., radiation counters, radio noise, audio noise, thermal noise in diodes, leaky capacitors, mercury discharge tubes etc
- Starting to see such h/w in new CPU's
- Problems of **bias** or uneven distribution in signal
  - Have to compensate for this when sample, often by passing bits through a hash function
  - Best to only use a few noisiest bits from each sample
  - RFC4086 recommends using multiple sources + hash

Stream Ciphers

- Process message bit by bit (as a stream)
- A pseudo random **keystream** XOR'ed with plaintext bit by bit

$$C_i = M_i \text{ XOR StreamKey}_i$$

- But must never reuse stream key otherwise messages can be recovered



RC4

- A proprietary cipher owned by RSA DSI
- Another Ron Rivest design, simple but effective
- Variable key size, byte-oriented stream cipher
- Widely used (web SSL/TLS, wireless WEP/WPA)
- Key forms random permutation of all 8-bit values
- Uses that permutation to scramble input info processed a byte at a time

RC4 Key Schedule

- Start with an array S of numbers: 0..255
- Use key to well and truly shuffle
- S forms **internal state** of the cipher

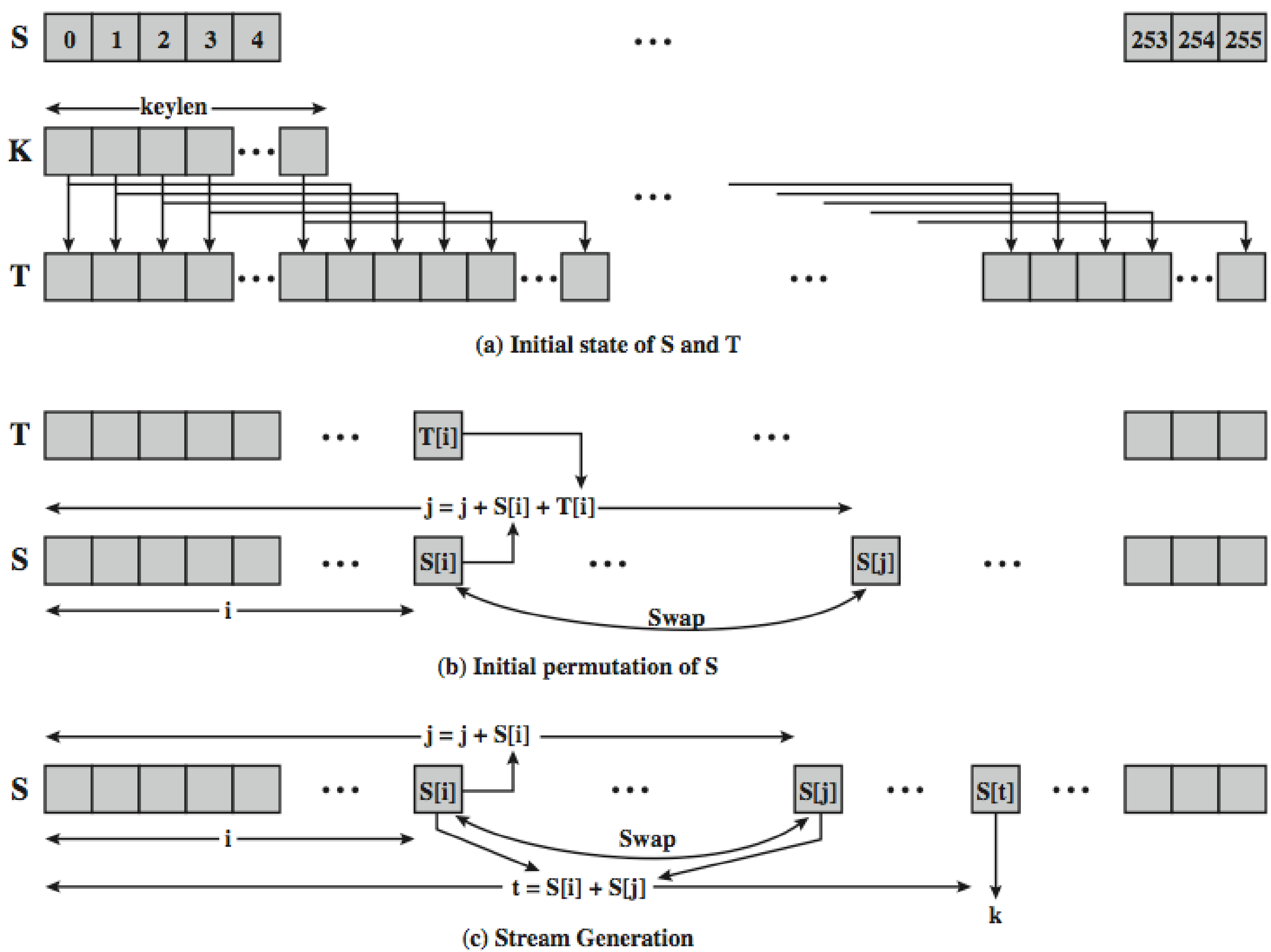
```
for i = 0 to 255 do
    S[i] = i
    T[i] = K[i mod keylen])
j = 0
for i = 0 to 255 do
    j = (j + S[i] + T[i]) (mod 256)
    swap (S[i], S[j])
```

RC4 Encryption

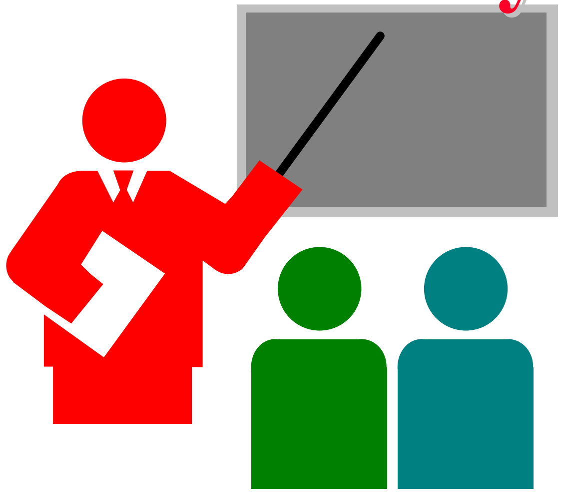
- Encryption continues shuffling array values
- Sum of shuffled pair selects "stream key" value from permutation
- XOR S[t] with next byte of message to en/decrypt

```
i = j = 0
for each message byte M_i
    i = (i + 1) (mod 256)
    j = (j + S[i]) (mod 256)
    swap(S[i], S[j])
    t = (S[i] + S[j]) (mod 256)
    C_i = M_i XOR S[t]
```

RC4 Overview



Summary



1. Pseudorandom number generators use a seed and a formula to generate the next number
2. Stream ciphers xor a random stream with the plain text.
3. RC4 is a stream cipher

Homework 7

a. Find the period of the following generator using seed x_0=1:

$$x_n = 5x_{n-1} \bmod 2^5$$

b. Now repeat part a with seed x_0 =2

c. What RC4 key value will leave S unchanged during initialization? That is, after the initial permutation of S, the entries of S will be equal to the values from 0 through 255 in ascending order.