Random Bit Generation and Stream Ciphers

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Audio/Video recordings of this lecture are available at:

http://www.cse.wustl.edu/~jain/cse571-17/

1. Principles of Pseudorandom Number Generation
2. Pseudorandom number generators
3. Pseudorandom number generation using a block cipher
4. Stream Cipher
5. RC4

These slides are based on Lawrie Brown's slides supplied with William Stalling's book "Cryptography and Network Security: Principles and Practice," 7th Ed, 2017.

Pseudo Random Numbers

- Many uses of **random numbers** in cryptography
  - Nonces in authentication protocols to prevent replay
  - Keystream for a one-time pad
- These values should be
  - Statistically random, uniform distribution, independent
  - Unpredictability of future values from previous values
- True random numbers provide this
- Psuedo $\Rightarrow$ Deterministic, reproducible, generated by a formula

A Sample Generator

$$x_n = f(x_{n-1}, x_{n-2}, \ldots)$$

- For example,

  $$x_n = 5x_{n-1} + 1 \mod 16$$

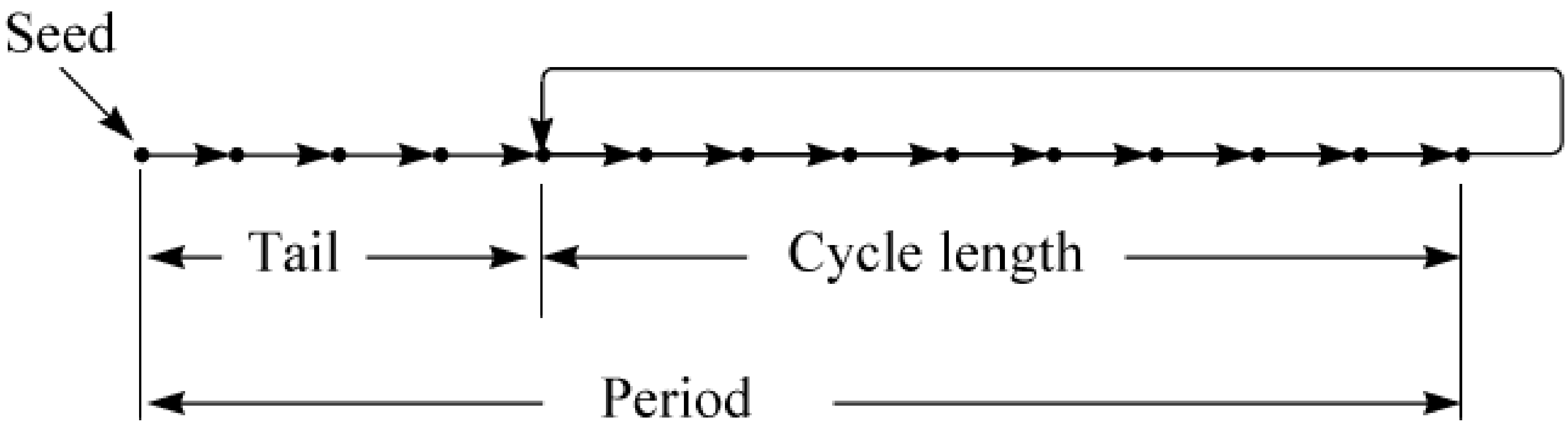
- Starting with x_0=5:

  $$x_1 = 5(5) + 1 \mod 16 = 26 \mod 16 = 10$$

- The first 32 numbers obtained by the above procedure 10, 3, 0, 1, 6, 15, 12, 13, 2, 11, 8, 9, 14, 7, 4, 5 10, 3, 0, 1, 6, 15, 12, 13, 2, 11, 8, 9, 14, 7, 4, 5.
- By dividing x's by 16:
  0.6250, 0.1875, 0.0000, 0.0625, 0.3750, 0.9375, 0.7500, 0.8125, 0.1250, 0.6875, 0.5000, 0.5625, 0.8750, 0.4375, 0.2500, 0.3125, 0.6250, 0.1875, 0.0000, 0.0625, 0.3750, 0.9375, 0.7500, 0.8125, 0.1250, 0.6875, 0.5000, 0.5625, 0.8750, 0.4375, 0.2500, 0.3125.

Terminology

- **Seed** = x_0
- **Pseudo-Random**: Deterministic yet would pass randomness tests
- Fully Random: Not repeatable
- **Cycle length**, **Tail**, **Period**



Linear-Congruential Generators

- Discovered by D. H. Lehmer in 1951
- The residues of successive powers of a number have good randomness properties.

$$x_n = a^n \bmod m$$

Equivalently,

$$x_n = a x_{n-1} \bmod m$$

a = multiplier

m = modulus

Linear-Congruential Generators (Cont)

- Lehmer's choices: $a = 23$ and $m = 10^8+1$
- Good for ENIAC, an 8-digit decimal machine.
- Generalization:

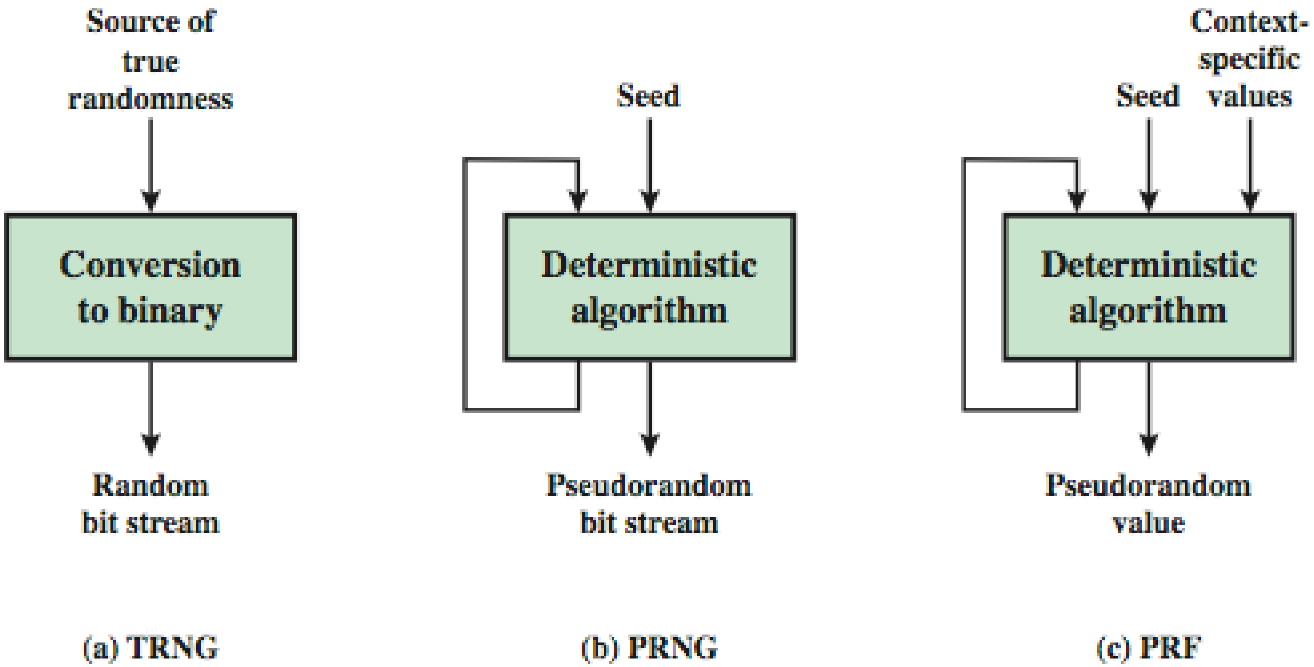
$$x_n = ax_{n-1} + b \bmod m$$

- Can be analyzed easily using the theory of congruences
  $\Rightarrow$ Mixed Linear-Congruential Generators
  or Linear-Congruential Generators (LCG)
- Mixed = both multiplication by a and addition of b

Blum Blum Shub Generator

- Use least significant bit from iterative equation:
  - $x_i = x_{i-1}^2 \bmod n$
  - where $n = p.q$, and primes p, q
  - p mod 4=3, q mod 4=3
  - E.g., p=7, q=11
- Unpredictable, passes **next-bit** test
  ⇒ Cannot predict (k+1)st bit given k bits with probability greater than ½.
- Security rests on difficulty of factoring n
- Is unpredictable given any run of bits
- Slow, since very large numbers must be used
- Too slow for cipher use, good for key generation

Random & Pseudorandom Number Generators



Continuous Stream (stream cipher)

Fixed Length # (Keys, Nonces)

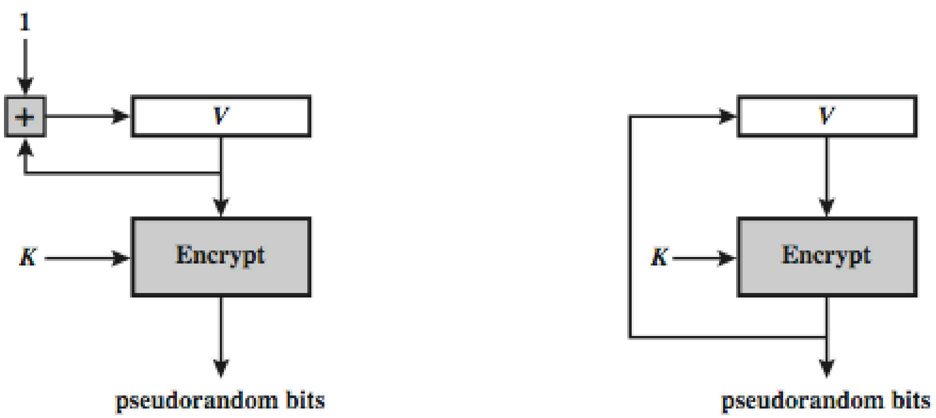
Using Block Ciphers as PRNGs

- Can use a block cipher to generate random numbers for cryptographic applications,
- For creating session keys from master key
- CTR (Counter Mode)

$$X_i = E_K[V_i]$$

- OFB (Output Feedback)

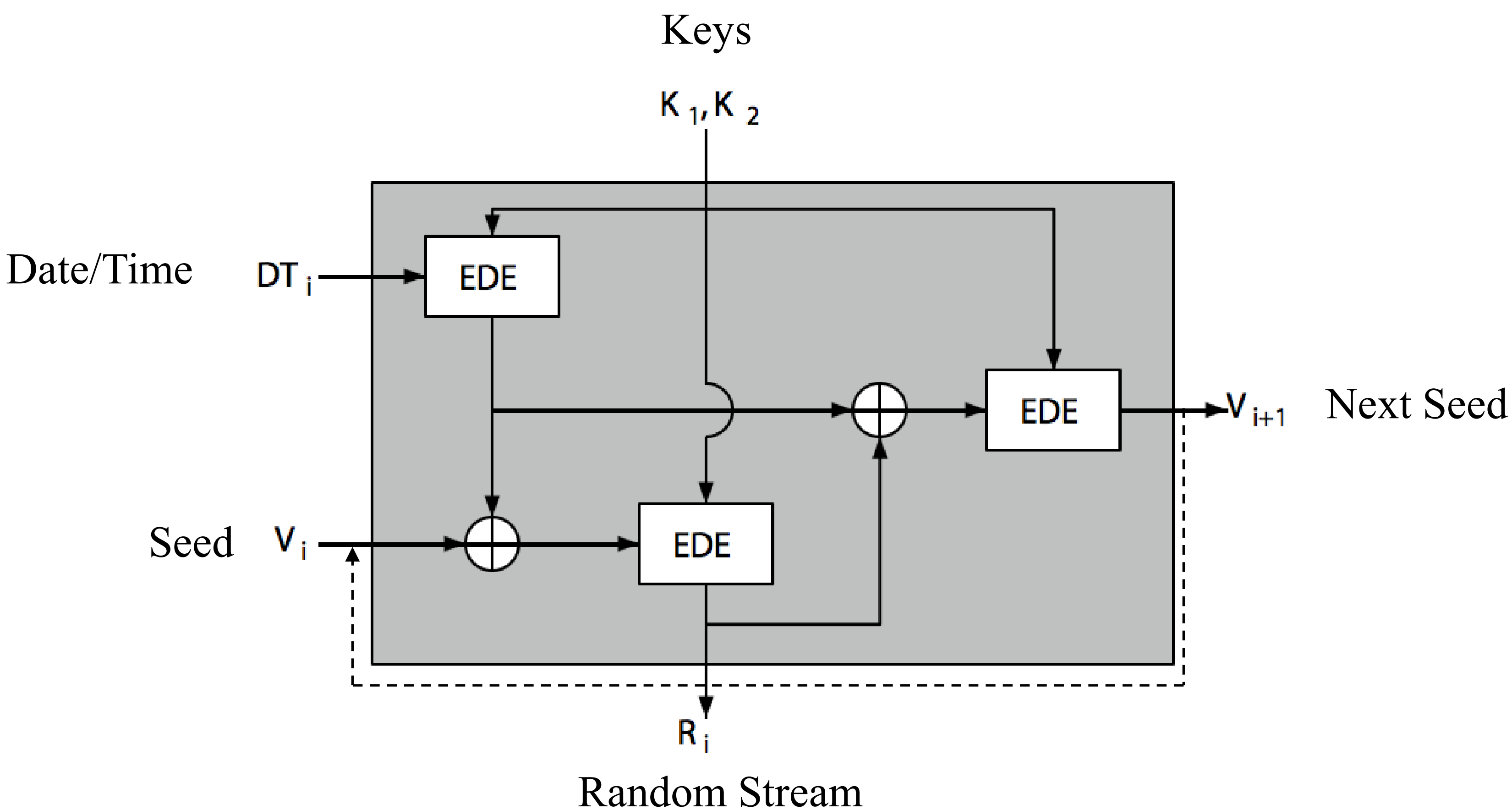
$$X_i = E_K[X_{i-1}]$$



(a) CTR Mode

(b) OFB Mode

ANSI X9.17 PRG



EDE= Triple DES

=Encrypt using k_1+Decrypt using k_2+Encrypt using k_1

Natural Random Noise

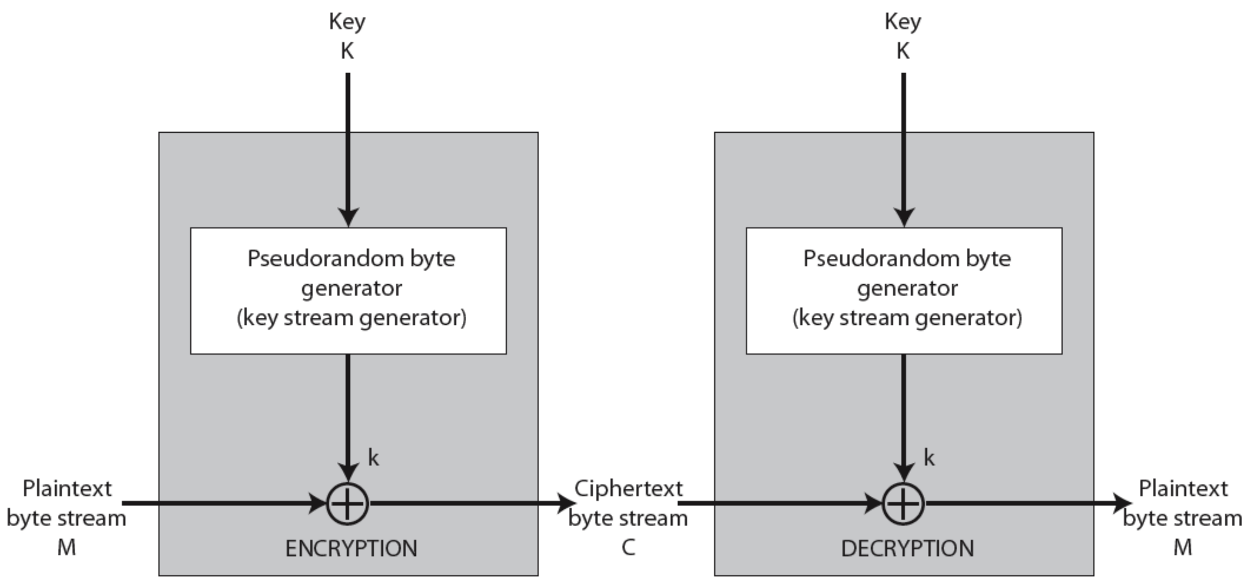
- Best source is natural randomness in real world
- Find a regular but random event and monitor
- Do generally need special h/w to do this
  - E.g., radiation counters, radio noise, audio noise, thermal noise in diodes, leaky capacitors, mercury discharge tubes etc
- Starting to see such h/w in new CPU's
- Problems of **bias** or uneven distribution in signal
  - Have to compensate for this when sample, often by passing bits through a hash function
  - Best to only use a few noisiest bits from each sample
  - RFC4086 recommends using multiple sources + hash

Stream Ciphers

- Process message bit by bit (as a stream)
- A pseudo random **keystream** XOR'ed with plaintext bit by bit

$$C_i = M_i \text{ XOR StreamKey}_i$$

- But must never reuse stream key otherwise messages can be recovered



RC4

- A proprietary cipher owned by RSA
- Another Ron Rivest design, simple but effective
- Variable key size, byte-oriented stream cipher
- Widely used (web SSL/TLS, wireless WEP/WPA)
- Key forms random permutation of all 8-bit values
- Uses that permutation to scramble input info processed a byte at a time

RC4 Initialization

- Start with an array S of numbers: 0..255
- S forms **internal state** of the cipher

  for i = 0 to 255 do

  S[i] = i

  T[i] = K[i mod keylen]

S	0	1	2		254	255
T	k_0	k_1	k_2		k_{254}	k_{255}

If key is shorter than 256 bytes, it is simply repeated to make 256 bytes.

RC4 Initial Permutation

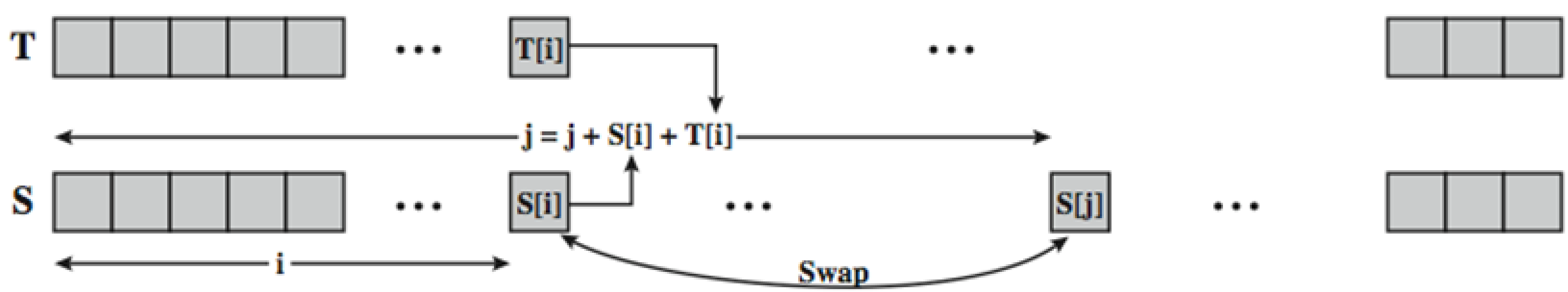
- Use key to well and truly shuffle

  j = 0

  for i = 0 to 255 do

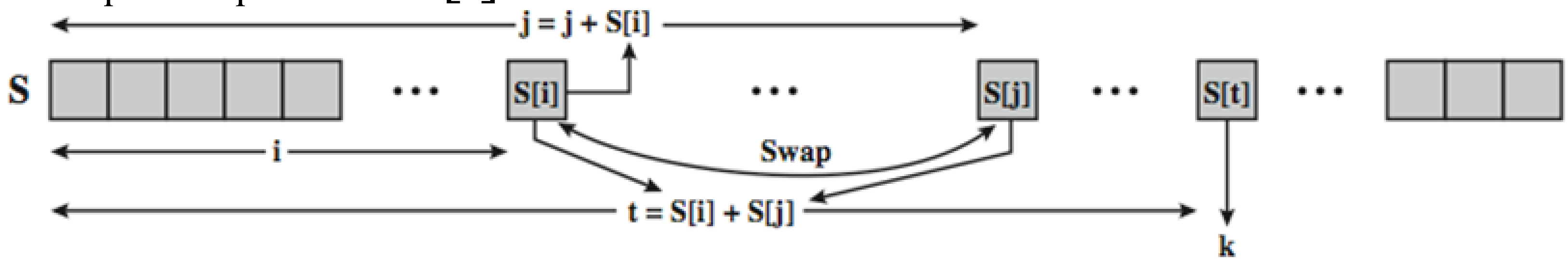
      j = (j + S[i] + T[i]) (mod 256)

      swap (S[i], S[j])

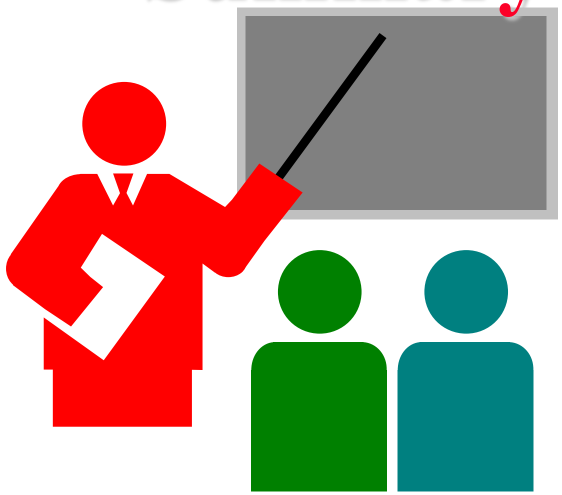


RC4 Encryption

- Encryption continues shuffling array values
- Sum of shuffled pair selects "stream key" value from permutation
- $i = j = 0$
  $i = (i + 1) \pmod{256}$
  $j = (j + S[i]) \pmod{256}$
  swap(S[i], S[j])
  $t = (S[i] + S[j]) \pmod{256}$
  Key=S[t]
  $C_i = M_i$ XOR S[t]



Summary



1. Pseudorandom number generators use a seed and a formula to generate the next number
2. Stream ciphers xor a random stream with the plain text.
3. RC4 is a stream cipher

Homework 8

a. Find the period of the following generator using seed x_0=1:

$$x_n = 5x_{n-1} \bmod 2^5$$

b. Now repeat part a with seed x_0 =2

c. What RC4 key value will leave S unchanged during initialization? That is, after the *initial permutation* of S, the entries of S will be equal to the values from 0 through 255 in ascending order.

Acronyms

- AES Advanced Encryption Standard
- ANSI American National Standards Institute
- BBS Blum, Blum, Shub
- CPU Central Processing Unit
- CSPRBG Cryptographically Secure
- CTR Counter
- DES Data Encryption Standard
- EDE Encrypt-Decrypt-Encrypt
- ENIAC An 8-digit decimal machine.
- ID Identifier
- LAN Local Area Networks
- LCG Linear-Congruential Generator
- MD5 Message Digest 5
- OFB Output Feedback
- OFV Output Feedback Value
- PRBG Pseudorandom bit generator

Acronyms (Cont)

PRF	Pseudorandom function
PRG	Pseudorandom Generator
RC4	Ron's Code 4
RF	Request for Comment
RSA	Rivest, Samir, and Adleman
SHA	Secure Hash Algorithm
SP	Standard Protocol
SSL	Secure Socket Layer
TLS	Transport Layer Security
TRNG	True random number generator
WEP	Wired equivalent privacy
WPA	Wi-Fi Protected Access
XOR	Exclusive-Or

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Related Modules

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CSE473S: Introduction to Computer Networks (Fall 2016),
http://www.cse.wustl.edu/~jain/cse473-16/index.html

Wireless and Mobile Networking (Spring 2016),
http://www.cse.wustl.edu/~jain/cse574-16/index.html

CSE571S: Network Security (Fall 2014),
http://www.cse.wustl.edu/~jain/cse571-14/index.html

Audio/Video Recordings and Podcasts of
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