

Digital Signature



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Audio/Video recordings of this lecture are available at:

<http://www.cse.wustl.edu/~jain/cse571-17/>

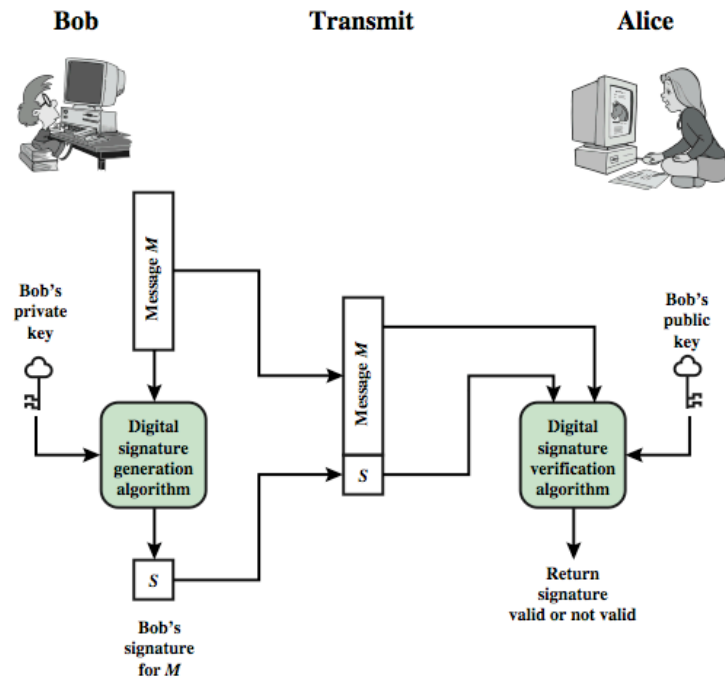


1. Digital Signatures
2. ElGamal Digital Signature Scheme
3. Schnorr Digital Signature Scheme
4. Digital Signature Standard (DSS)

These slides are based partly on Lawrie Brown's slides supplied with William Stallings's book "Cryptography and Network Security: Principles and Practice," 7th Ed, 2017.

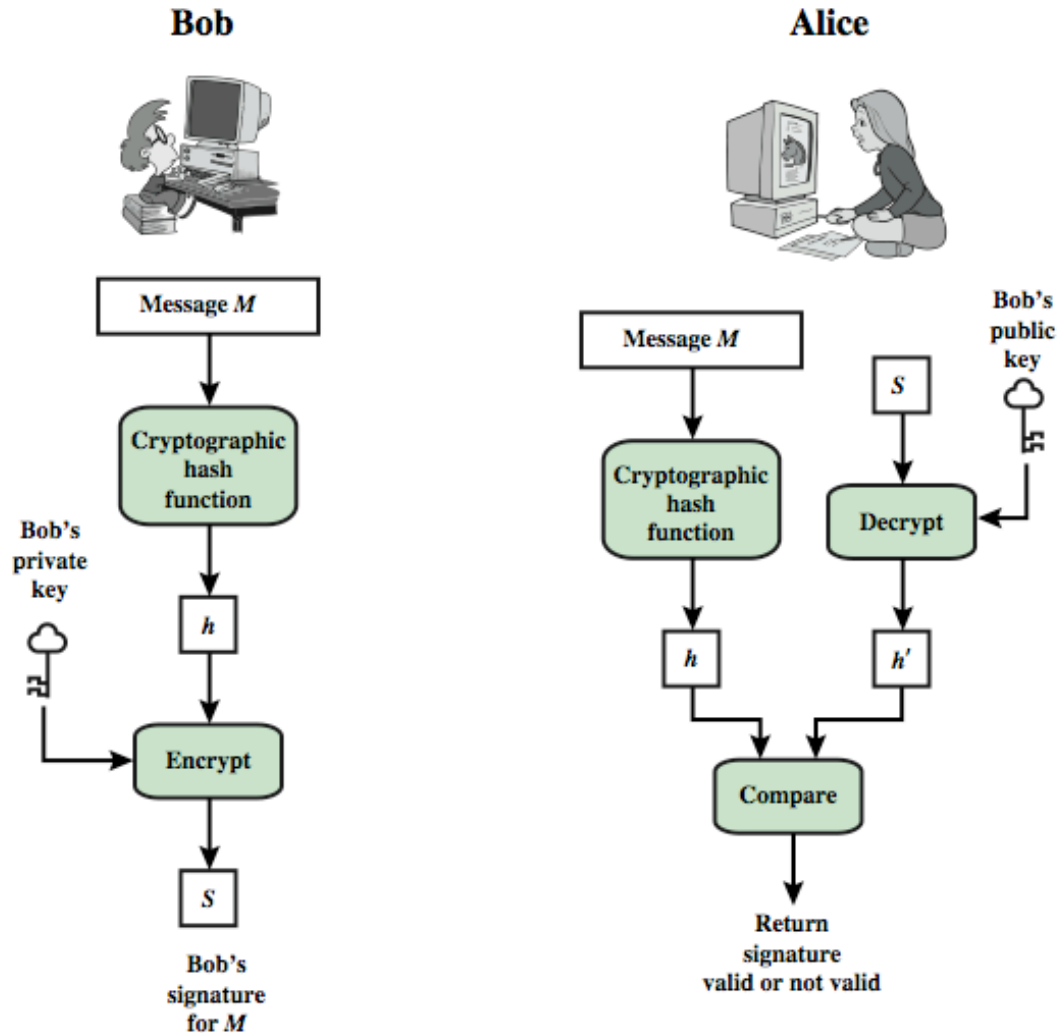
Digital Signatures

- ❑ Verify author, date & time of signature
- ❑ Authenticate message contents
- ❑ Can be verified by third parties to resolve disputes



Ref: <http://en.wikipedia.org/wiki/Non-repudiation>, http://en.wikipedia.org/wiki/Digital_signature,
http://en.wikipedia.org/wiki/Digital_signatures_and_law

Digital Signature Model



Attacks

- ❑ In the order of Increasing severity.
- ❑ C=Attacker, A=Victim
- 1. **Key-only attack**: C only knows A's public key
- 2. **Known message attack**: C has a set of messages, signatures
- 3. **Generic chosen message attack**: C obtains A's signatures on messages selected without knowledge of A's public key
- 4. **Directed chosen message attack**: C obtains A's signatures on messages selected after knowing A's public key
- 5. **Adaptive chosen message attack**: C may request signatures on messages depending upon previous message-signature pairs

Forgeries

1. **Total break:** C knows A's private key
2. **Universal forgery:** C can generate A's signatures on any message
3. **Selective forgery:** C can generate A's signature for a particular message chosen by C
4. **Existential forgery:** C can generate A's signature for a message not chosen by C

Digital Signature Requirements

- ❑ Must depend on the message signed
- ❑ Must use information unique to sender
 - To prevent both forgery and denial
- ❑ Must be relatively easy to produce
- ❑ Must be relatively easy to recognize & verify
 - Directed \Rightarrow Recipient can verify
 - Arbitrated \Rightarrow Anyone can verify
- ❑ Be computationally infeasible to forge
 - With new message for existing digital signature
 - With fraudulent digital signature for given message
- ❑ Be able to retain a copy of the signature in storage

ElGamal Digital Signatures

- ❑ Signature variant of ElGamal, related to D-H
 - Uses exponentiation in a finite (Galois) field
 - Based on difficulty of computing discrete logarithms, as in D-H
- ❑ Each user (e.g., Alice) generates her key
 - Given a large prime q and its primitive root a
 - Alice chooses a private key: $1 < x_A < q-1$
 - Alice computes her **public key**: $y_A = a^{x_A} \bmod q$

ElGamal Digital Signature

- Alice signs a message M to Bob by computing
 - Hash $m = H(M)$, $0 \leq m \leq (q-1)$
 - Choose a random integer K with $1 \leq K \leq (q-1)$ and $\gcd(K, q-1)=1$ (K is the per message key)
 - Compute $S_1 = a^K \text{ mod } q$
 - Compute K^{-1} the inverse of $K \text{ mod } (q-1)$
 - Compute the value: $S_2 = K^{-1}(m - x_A S_1) \text{ mod } (q-1)$
 - If S_2 is zero, start with a new K
 - Signature is: (S_1, S_2)
- Any user B can verify the signature by computing
 - $V_1 = a^m \text{ mod } q$
 - $V_2 = y_A^{S_1} S_1^{S_2} \text{ mod } q$
 - Signature is valid if $V_2 = V_1$

$$(a^{x_A})^{S_1} (a^K)^{S_2} = a^{x_A S_1 + K S_2} = a^{x_A S_1 + m - x_A S_1} = a^m$$

ElGamal Signature Example

- ❑ GF(19) $q=19$ and $a=10$
- ❑ Alice computes her key:
 - A chooses $x_A=16$ & computes $y_A=10^{16} \bmod 19 = 4$
- ❑ Alice signs message with hash $m=14$ as (3,4):
 - Choosing random $K=5$ which has $\gcd(18,5)=1$
 - Computing $S_1 = 10^5 \bmod 19 = 3$
 - Finding $K^{-1} \bmod (q-1) = 5^{-1} \bmod 18 = 11$
 - Computing $S_2 = 11(14-16 \times 3) \bmod 18 = 4$
- ❑ Any user B can verify the signature by computing
 - $V_1 = a^m \bmod q = 10^{14} \bmod 19 = 16$
 - $V_2 = y_A^{S_1} S_1^{S_2} \bmod q = 4^3 \times 3^4 = 5184 \bmod 19 = 16$
 - Since $16 = 16$, the signature is valid

Schnorr Digital Signatures

- ❑ Also uses exponentiation in a finite (Galois) field
- ❑ Minimizes message dependent computation
 - Main work can be done in idle time
- ❑ Using a prime modulus p
 - $p-1$ has a prime factor q of appropriate size
 - typically p 1024-bit and q 160-bit (SHA-1 hash size)
- ❑ Schnorr Key Setup: Choose suitable primes p, q
 - Choose a such that $a^q = 1 \pmod p$
 - $\{a, p, q\}$ are global parameters for all
 - Each user (e.g., A) generates a key
 - Chooses a secret key (number): $0 < s < q$
 - Computes his **public key**: $v = a^{-s} \pmod q$

Schnorr Signature

- ❑ User signs message by
 - Choosing random r with $0 < r < q$ and computing $x = a^r \bmod p$
 - Concatenating message with x and hashing:
$$e = H(M \parallel x)$$
 - Computing: $y = (r + se) \bmod q$
 - Signature is pair (e, y)
- ❑ Any other user can verify the signature as follows:
 - Computing: $x' = a^y v^e \bmod p$
 - Verifying that: $e = H(M \parallel x')$
 - $x' = a^y v^e = a^y a^{-se} = a^{y-se} = a^r = x \bmod p$

Ref: http://en.wikipedia.org/wiki/Schnorr_signature

Digital Signature Standard (DSS)

- ❑ US Govt approved signature scheme
- ❑ Designed by NIST & NSA in early 90's
- ❑ Published as FIPS-186 in 1991
- ❑ Revised in 1993, 1996 & then 2000
- ❑ Uses the SHA hash algorithm
- ❑ DSS is the standard, DSA is the algorithm
- ❑ FIPS 186-2 (2000) includes alternative RSA & elliptic curve signature variants
- ❑ DSA is digital signature only

Ref: http://en.wikipedia.org/wiki/Digital_Signature_Algorithm

Digital Signature Algorithm (DSA)

- ❑ Creates a 320 bit signature
- ❑ With 512-1024 bit security
- ❑ Smaller and faster than RSA
- ❑ A digital signature scheme only
- ❑ Security depends on difficulty of computing discrete logarithms
- ❑ Variant of ElGamal & Schnorr schemes

DSA Key Generation

- ❑ Shared global public key values (p, q, g):
 - Choose 160-bit prime number q
 - Choose a large prime p with $2^{L-1} < p < 2^L$
 - ❑ Where $L = 512$ to 1024 bits and is a multiple of 64
Now extended to 2048 or 3072 bits
 - ❑ Such that q is a 160 bit prime divisor of $(p-1)$
 - Choose $g = h^{(p-1)/q}$
 - ❑ Where $1 < h < p-1$ and $h^{(p-1)/q} \bmod p > 1$
Commonly $h=2$ is used
- ❑ Users choose private & compute public key:
 - Choose random private key: $x < q$
 - Compute public key: $y = g^x \bmod p$

DSA Signature Creation

- ❑ To **sign** a message M the sender:

- Generates a random signature key k , $k < q$
- Note: k must be random, be destroyed after use, and never be reused

- ❑ Then computes signature pair:

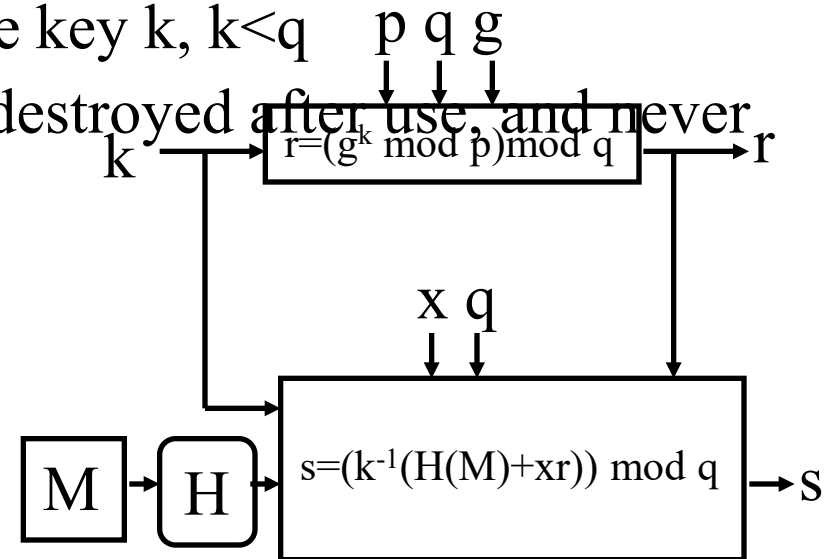
$$r = (g^k \bmod p) \bmod q$$

if $r=0$ choose another k

$$s = [k^{-1}(H(M) + xr)] \bmod q$$

if $s=0$ choose another k

- ❑ Sends signature (r,s) with message M



DSA Signature Verification

- To **verify** a signature, recipient computes:

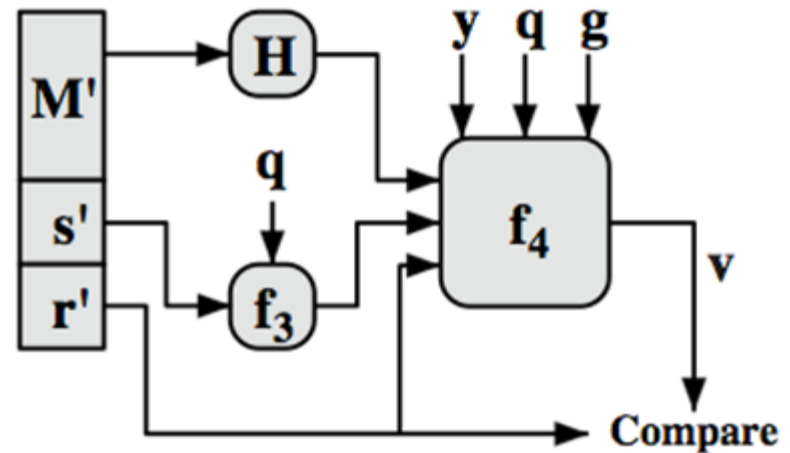
$$w = s^{-1} \text{ mod } q$$

$$u1 = [H(M)w] \text{ mod } q$$

$$u2 = (rw) \text{ mod } q$$

$$v = [(g^{u1} y^{u2}) \text{ mod } p] \text{ mod } q$$

- If $v=r$ then signature is verified

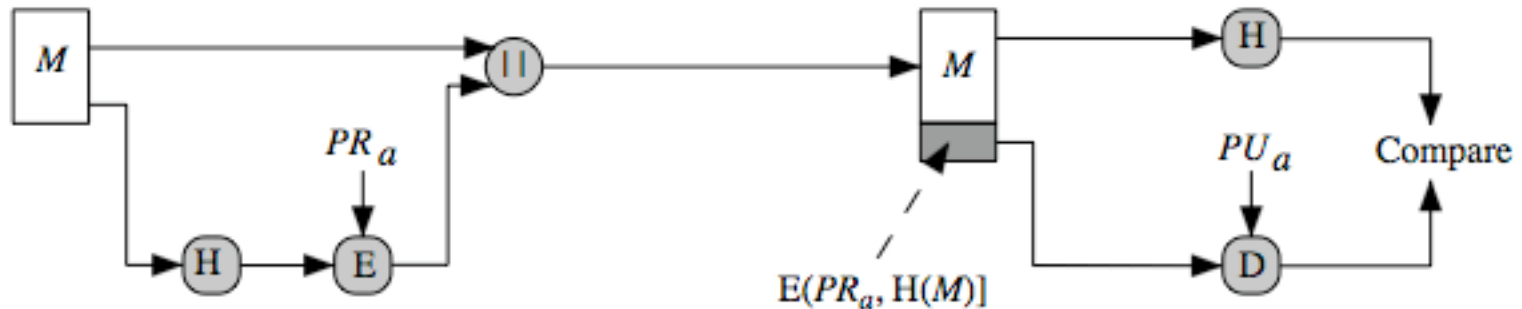


$$w = f_3(s', q) = (s')^{-1} \text{ mod } q$$

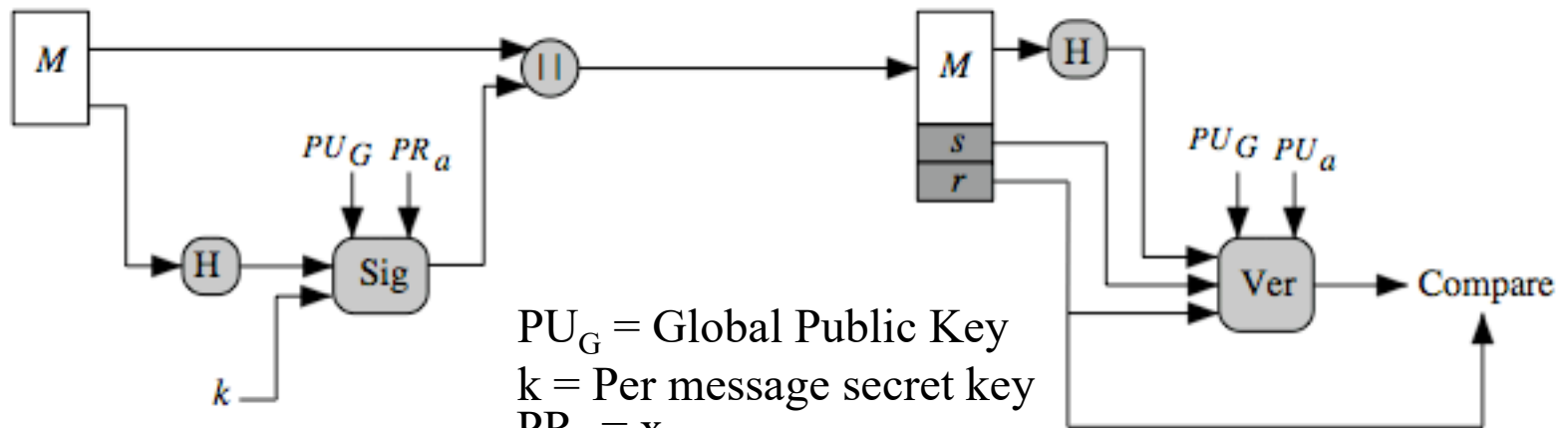
$$v = f_4(y, q, g, H(M'), w, r')$$

$$= ((g^{H(M')w} \text{ mod } q \cdot y^{r'w} \text{ mod } q) \text{ mod } p) \text{ mod } q$$

DSS vs. RSA Signatures



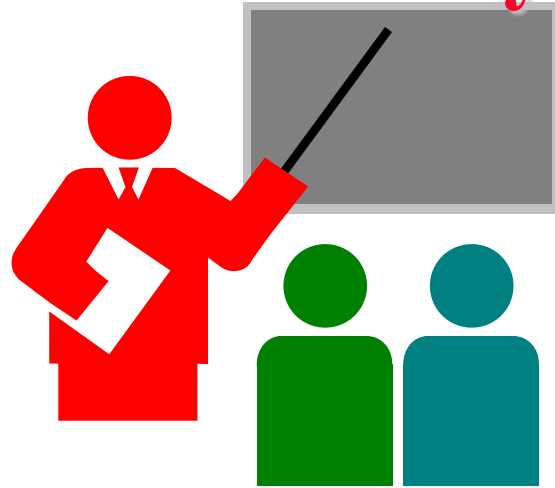
(a) RSA Approach



PU_G = Global Public Key
 k = Per message secret key
 $PR_a = x$

(b) DSS Approach

Summary



1. Digital signature depends upon the message and some information unique to the signer to prevent forgery and denial. Anyone should be able to verify.
2. ElGamal/Schnorr/DSA signatures use a per-message secret key and are based on exponentiation
3. DSA produces a 320 bit signature

Homework 13

- ❑ DSA specifies that if signature generation process results in a value of $s=0$, a new value of k should be generated and the signature should be recalculated. Why?
- ❑ Suppose Alice signed a message M using DSA with a specific k value and then the k value was compromised. Can Alice still use her private key for future digital signatures?
- ❑ Hint: Show that the private key of the signer can be easily computed in both of the above cases.

Lab 13: Nessus

- ❑ In this lab, you will use Nessus software to scan vulnerabilities in your network
- ❑ Nessus is a common vulnerability scanning platform for auditors and security analysts.
- ❑ You can download Nessus from <https://www.tenable.com/products/nessus/select-your-operating-system>
- ❑ You can do the lab on windows, Mac, or Kali (might not work from USB boot)

Ref: [https://en.wikipedia.org/wiki/Nessus_\(software\)](https://en.wikipedia.org/wiki/Nessus_(software))

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Lab 13 (Cont)

- ❑ Download and start Nessus on your system
 - For windows and Mac, it should be straightforward
 - For Kali
 - ❑ `dpkg -i your_downloaded_file`
 - ❑ `/etc/init.d/nessusd start`
- ❑ Go to <https://localhost:8834> and expect the exception this time
- ❑ Login and setup your account
 - Select “use at HOME” option when setting your account
 - Access your activation code from <http://www.tenable.com/products/nessus/nessus-plugins/obtain-an-activation-code>

Lab 13 (Cont)

- ❑ Start a new scan and choose advance scan
 - Name = Name your
 - ScanType = Run Now (drop down options)
 - Policy = Internal Network Scan (drop down options)
 - Targets = Your Internal home IP range.
- ❑ Start the scan and wait until the scan finishes
- ❑ Submit a screenshots of host, remediation (if you have) and history
- ❑ Choose a specific host (not yours)
 - Submit vulnerabilities
- ❑ Uninstall

Acronyms

- ❑ DSA Digital Signature Algorithm
- ❑ DSS Digital Signature Standard
- ❑ FIPS Federal Information Processing Standard
- ❑ GF Galois Field
- ❑ MIME Multipurpose Internet Mail Extension
- ❑ NIST National Institute of Technology
- ❑ NSA National Security Agency
- ❑ PR Private Key
- ❑ PU Public Key
- ❑ RSA Rivest, Shamir, and Adleman
- ❑ SHA Secure Hash Algorithm

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Related Modules



CSE571S: Network Security (Spring 2017),
<http://www.cse.wustl.edu/~jain/cse571-17/index.html>

CSE473S: Introduction to Computer Networks (Fall 2016),
<http://www.cse.wustl.edu/~jain/cse473-16/index.html>



Wireless and Mobile Networking (Spring 2016),
<http://www.cse.wustl.edu/~jain/cse574-16/index.html>

CSE571S: Network Security (Fall 2014),
<http://www.cse.wustl.edu/~jain/cse571-14/index.html>



Audio/Video Recordings and Podcasts of
Professor Raj Jain's Lectures,
<https://www.youtube.com/channel/UCN4-5wzNP9-ruOzQMs-8NUw>