CSE 240, Fall, 2013 Homework 2
Due, Tuesday September 17. Can turn in class, at the beginning of class, or earlier in the mailbox labelled “Pless” in Bryan Hall, room 509c.

Practice Problems:

1. Consider the following english arguments. Define propositions/predicates and translate these arguments into logic, then prove or disprove whether the form of the argument is valid.

(a) All Computer Science majors are people.
Some computer science majors are logical thinkers

Some people are logical thinkers.

(b) If I like mathematics then I will study.
Either i don’t study or I pass mathematics
If I don’t pass mathematics, then I don’t graduate.

If I graduate, then I like mathematics.

2. Using (and citing) rules of inference and logical equivalence, Prove the following form of argument is valid:

\[ a \lor b \]
\[ a \rightarrow p \]
\[ b \rightarrow p \]

\[ p \]

3. Prove \( \forall a, b, c, abc \text{ is odd} \rightarrow a \text{ is odd and } b \text{ is odd and } c \text{ is odd}. \)

4. Challenge Problem: Let \( P(x) \) denote that \( x \) is a politician, and \( Q(x, y) \) denote that \( x \) quotes \( y \) where the universe of discourse for \( x \) and \( y \) are all people. Express each of the below statements using these predicates, quantifiers, and logical connectives.

(a) Every politician quotes someone, but no politician is quoted by every one of the politicians that she quotes.

(b) If a politician quotes any politician that does not quote him, he quotes every politician that quotes no politician.

5. A real number \( x \) is an upper bound of a set \( S \) of real numbers if \( x \) is greater than or equal to every number of \( S \).

(a) Use quantifiers to express the fact that \( x \) is an upper bound of \( S \). That is, define the proposition \( \text{UB}(x) \) that is true when \( x \) is an upper bound of \( S \).

(b) New definition: A real number \( x \) is called the least upper bound of a set \( S \) of real numbers if \( x \) is an upper bound of \( S \), and \( x \) is less than or equal to every upper bound of \( S \).

Use quantifiers to express the fact that \( x \) is a least upper bound of \( S \).

(c) Define any set \( S \) of real numbers which has a least upper bound that is not in the set \( S \).

(d) Prove: \( \forall n, n \mod 3 == 2 \rightarrow n \text{ is not a perfect square} \)
Problems to turn in

1. Using (and citing) rules of inference and logical equivalence, Prove the following form of argument is valid:

\[(P \land Q) \lor R \quad R \rightarrow S \quad \therefore P \lor S\]

2. Prove or disprove (for positive integers): \(\forall a, b, c, \) if \(a|b\) and \(b|c\) then \(a|(b + c)\)

3. Prove or disprove (for positive integers): \(\forall a, b, c, d,\) if \(ab|cd\) then \(a|c\) or \(a|d\).