Homework 2

Solutions

September 17

1. Using (and citing) rules of inference and logical equivalence, prove the following form of argument is valid:

\[(P \land Q) \lor R \quad R \rightarrow S \quad \therefore P \lor S\]

There are many possible solutions. Here is one:

1. \((P \land Q) \lor R\) Given
2. \(R \rightarrow S\) Given
3. \(\neg R \lor S\) Implies Rule
4. \(R \lor (P \land Q)\) 1, Commutative
5. \((S \lor (P \land Q))\) 3, 4, Resolution
6. \((S \lor P) \land (S \lor Q)\) 5, Distributive
7. \((S \lor P)\) 6, Simplification
8. \(P \lor S\) 7, Commutative

2. Prove or disprove (for positive integers): \(\forall a, b, c,\) if \(a|b\) and \(b|c\) then \(a|(b + c)\)

Proof. Assume \(a|b\) and \(b|c\). The goal is to prove \(a|(b + c)\).

Since \(a|b, b = ak\) for some integer \(k\) by definition of “divides”. Similarly, since \(b|c, c = bn\) for some integer \(n\).

\[
\begin{align*}
  c &= bn \\
  c &= (ak)n \\
  b + c &= ak + akn \\
  b + c &= a(k + kn)
\end{align*}
\]

Since \(k + kn\) is an integer, \(a|(b + c)\).

3. Prove or disprove (for positive integers): \(\forall a, b, c,\) if \(ab|cd\) then \(a|c\) or \(a|d\).

This proposition is false. Here is one counterexample.

Choose \(a = 9, b = 4, c = 6,\) and \(d = 6\).

Then \(ab = 36, cd = 36,\) and therefore \(ab|cd\). But, \(a \not| c\) and \(a \not| d\), because \(9 \not| 6\) in each case.