Practice Problems:

1. The city of Logicus is inhabited by citizens with the following characteristics:

   1. The citizens are at a party where they can see everyone’s face except their own;
   2. No citizen ever tells another anything that would embarrass that citizen (ie, they don’t talk about the poppy seeds to each other)
   3. All citizens have perfect (and instantaneous) logical reasoning abilities.

Brian crashes the party and announces that at least one person has poppy seeds in their teeth. After his proclamation, Brian immediately leaves. The citizens (not having a mirror) decide that every 5 minutes (where all citizens look at the same clock) whoever discovers they have poppy seeds in their teeth will excuse themselves from the party (and we’ll assume that nobody would ever dream of leaving the party for any other reason).

In summary, we have: nobody directly knows if they have poppy seeds in their teeth, because nobody would tell them this; however, everyone can see who else has poppy seeds in their teeth.

Let \( n \) denote the number of people at the party who have poppy seeds in their teeth. (Remember that the citizens only know that \( n \geq 1 \) and are not directly given any other information about its value.)

(a) When, if ever, will those with poppy seeds on their teeth leave the party?

(b) Use a form of mathematical induction to prove that your answer is correct.

(c) If there is more than one person with poppy seeds, everyone already knew that some people have poppy seeds in their mouth. What did Brian actually do here?!

2. Prove for integers \( n \geq 0 \), \( 3|n^3 + 2n \)

3. Prove Bernoulli’s inequality:

\[
(1 + x)^n \geq 1 + nx
\]

for all non-negative integers \( n \), when \( x \) is a real number greater than -1.

Problems to turn in:

1. Use induction to prove the following for all integers \( n \geq 1 \).

\[
\frac{1 \cdot 3 \cdot 5 \cdots (2n + 1)}{2 \cdot 4 \cdot 6 \cdots (2n + 2)} \geq \frac{1}{2n + 2}
\]

2. Use induction to prove that you can create a knights tour of any grid that has \( n \times 3 \) squares, where \( n \) is even and \( n \geq 10 \). (Hint: YOU may need to use multiple base cases and/or strong induction). (One knight move corresponds to going two steps in one direction (up, down, left, or right) and 1 step in a perpendicular direction). A tour is a path that starts somewhere, and visits every square exactly once. It does not have to end where it starts.

3. Prove that \( \log_5 16 \) is irrational.