Practice Problems:

1. Use a loop invariant to prove that the following program is correct with respect to the initial assertion that \( x \) is a positive integer and the final assertion that \( ans = x^2 \).

   procedure square(x)
   
   \[ \begin{align*}
   i &= 1 \\
   j &= 1 \\
   \text{while } (i < x) \text{ do} \\
   & \quad j = j + 2i + 1 \\
   & \quad i = i + 1 \\
   \text{od} \\
   \text{return } j
   \end{align*} \]

2. Use a loop invariant to prove that the following program is correct with respect to the initial assertion that \( n \) is an integer \( \geq 0 \) and the final assertion that \( \text{palindrome}(n) \) returns the integer obtained by reversing the digits in \( n \). (For example, if \( n = 1432 \) then it should return 2341.). If it helps, you can use "Reverse(x)" in your logical arguments to represent the number that you would get by reversing the digits of \( x \).

   procedure palindrome(n)
   
   \[ \begin{align*}
   \text{reverse} &= 0 \\
   m &= n \\
   \text{while } m > 0 \\
   & \quad \text{temp} = m \% 10 \quad (\% \text{ is "mod", so } m \% 10 \text{ is } m \mod 10) \\
   & \quad \text{reverse} = \text{reverse} \times 10 + \text{temp} \\
   & \quad m = (m - \text{temp}) / 10 \\
   \text{return } \text{reverse}
   \end{align*} \]

Problems to turn in:

1. Use the idea of the “fast exponentiation” algorithm discussed in class to make a “faster multiplication” algorithm that returns the product of \( (x,y) \). Your program should be asymptotically faster than one that repetitively adds \( x \) to itself \( y \) times. (or \( y \) to itself \( x \) times). You are not allowed to multiply in your algorithm. You are permitted to divide by two. [as an aside, for binary numbers, this can be implemented as a simple bit shift, so it isn’t really cheating].

2. Define the following “Fibonacci-Like” recursive sequence:
   \[ a_1 = 1, a_2 = 5, \text{ and } \forall n \geq 3, a_n = 5a_{n-1} - 6a_{n-2} \]
   
   (a) Compute the terms \( a_3, a_4, a_5 \) (calculators are acceptable).
   (b) Prove by strong induction that \( \forall n \geq 1, a_n = 3^n - 2^n \)