(a) \# of marbles

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Move to Lose</td>
<td>TAKE 1</td>
<td>TAKE 2</td>
<td>TAKE 3</td>
<td>Lose</td>
<td>TAKE 1 (or 5)</td>
<td>Lose</td>
<td>TAKE 1 (or 5)</td>
<td>Lose</td>
<td>TAKE 2</td>
<td>lose</td>
<td>TAKE 2</td>
</tr>
</tbody>
</table>
| Player 1 can win if marbles can be written as \(3k+2\), \(3k+3\) or \(3k+4\)

(c) Proof: Base cases \(P(0), P(1)\)

Chart in (a) shows winning strategy for 2, 3, 4, 6, 7, 8 marbles so \(P(0) \land P(1)\) are true.

(d) Assume \(P(k)\) for \(P(k)\). Prove \(P(k+1)\)

How to play with

\(3(k+1)+2\) or
\(3(k+1)+3\) or
\(3(k+1)+4\) marbles?

Take 1 or 2 or 3 to leave \(3(k+1)+1\) marble.

Player 2 must take 1, 2, 3 or 5 marbles.

if they take 1, 2, 3, you win by hypothesis \(P(k)\)

if they take 5, you win by hypothesis \(P(k-1)\)

\(\therefore P(k+1)\) by induction
2a) \( x > 0 \)
\[
\begin{align*}
\{ & x' = x + 1 \\
\text{so } & x' > x \\
\text{and, by precondition } & x > 0 \\
\text{so } & x' > x > 0 \\
\text{so } & x > 0
\end{align*}
\]

After code \( x' = x + 1 \)

2b) if \( x = 1 \) then precondition holds, but \( x' = x - 1 \) gives \( x < 0 \) after loop so triple-noreturn.

2c) \( \{ \begin{align*}
\text{while } x \neq 0 \text{ do } \\
\text{\quad } x = x + 1 \\
\text{end}
\end{align*} \)
\[
\begin{align*}
\text{when loop ends, } x < 0 & \text{ by LI because loop ended} \\
\text{and } x = 0 & \\
\text{so } x = 0 & \text{ when loop ends, } \\
\text{ means the } x' \leq 0
\end{align*}
\]

2d) \( \{ \begin{align*}
\text{while } x \neq 0 \text{ do } \\
\text{\quad } x = x + 1 \\
\text{end}
\end{align*} \)
\[
\begin{align*}
\text{if the loop ends, } x = 0, \text{ but loop } \\
\text{may not end because if } x \text{ is positive } \\
\text{then } x \text{ will never become } 0
\end{align*}
\]