HW 7  ANSWERS

1. \[ R = \left\{ \left( (a,b), (c,d) \right) \mid ad = bc \right\} \]

(a) Prove \( R \) is reflexive.

\[ \forall x, y \quad \left( (x,x), (x,x) \right) \in R \]

\( xy = yx \) is true so

\( (x,y), (x,y) \in R \) for any \( (x,y) \)

(b) Prove \( R \) is symmetric

Assume \( \left( (a,b), (c,d) \right) \in R \)

Prove \( \left( (c,d), (a,b) \right) \in R \)

\( ad = bc \) by assumption, and defn of \( R \)

by rules of multiplication,

\( bc = cb \)

and \( ad = da \) so

\( cb = da \)

so \( \left( (c,d), (a,b) \right) \in R \)

so \( R \) is symmetric

(c) Prove \( R \) transitive:

Assume \( \left( (a,b), (c,d) \right) \in R \) and \( \left( (c,d), (e,f) \right) \in R \)

Prove \( \left( (a,b), (e,f) \right) \in R \)

\( ad = bc \) because \( (a,b), (c,d) \in R \)

\( \frac{a}{b} = \frac{c}{d} \) by math

also \( cf = de \) because \( (c,d), (e,f) \in R \)

so \( \frac{c}{d} = \frac{e}{f} \)
(c. cont.)
so \( \frac{a}{b} = \frac{e}{f} \)
and \( af = be \)
so \((a, b), (e, f) \in \mathcal{R}\)
so \(\mathcal{R}\) is transitive.

(b) Equiv classes are all sets of \((a, b)\) where the ratio \( \frac{a}{b} \)
in the same. For example \(\{(1,2), (2,4), (3,6), \ldots\}\)
is an equivalence class.

2. Prob (Flush | already have 5, 7 of hearts)

(a) \[ \begin{align*}
\frac{\binom{11}{3}}{\binom{50}{3}} &= 2 \quad \text{ways to pick 3 more hearts} \\
\end{align*} \]

(b) Prob (straight | 5, 7 of hearts)

\[ \begin{align*}
= \frac{3 \cdot 4^3}{\binom{50}{3}} &= 2 \quad \text{ways to pick any 3 more cards.} \\
\end{align*} \]

3 choices for starting point of the straight \cdot 4 choices for suit of each \(R\) of the 3 cards.
2 choices for any 3 more cards.

(c) Prob (straight-flush | 5, 7 of hearts)

\[ \begin{align*}
= \frac{3}{\binom{50}{3}} &= \text{choices for smallest value of the straight. [This is the only choice!]} \\
\end{align*} \]

(d) Prob (Flush in 7 cards | 5, 7 of hearts)

\[ \begin{align*}
= \frac{\binom{11}{3}(\binom{39}{2}) + \binom{11}{4}(\binom{39}{1}) + \binom{11}{5}}{\binom{50}{5}} \\
\end{align*} \]

3 options, get 3 more hearts and 2 non-hearts or 4 more hearts and 1 non-heart or 5 more hearts.